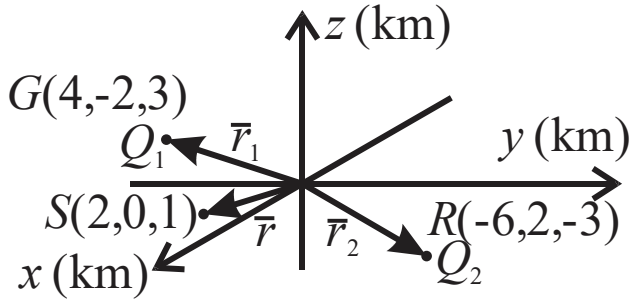
EE 381 Electric & Magnetic Fields Examination #2 (Fall 2026)

Name ghastly edition

Instructions: Place answers in indicated spaces. Use notation as given in class for coordinates and vectors. Use standard units. **Show all work** for full credit. Insert equation sheets inside exam.

1) In free space, the ring of Green Lantern is imbued with 15 μC and located at $G(4, -2, 3)$ while the ring of Red Lantern is charged with -20 μC and located at $R(-6, 2, -3)$. Calculate the absolute electrostatic potential V_S due to these rings at the sanctuary at $S(2, 0, 1)$. All spatial coordinates are in units of km.



Per (4.66), $V = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_k|} = \frac{Q_1}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_2|}$ for multiple point charges.

The field position vector is $\bar{r} = 2\hat{a}_x + \hat{a}_z$ (km).

Let $Q_1 = 15\ \mu$C & $\bar{r}_1 = 4\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ (km) and $Q_2 = $ -20 μC & $\bar{r}_2 = -6\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$ (km) for the point charges.

Then, the distances are:

$$|\bar{r} - \bar{r}_1| = |(2000 - 4000)\hat{a}_x + 2000\hat{a}_y + (1000 - 3000)\hat{a}_z| = |-2000\hat{a}_x + 2000\hat{a}_y - 2000\hat{a}_z|$$
$$= \sqrt{(-2000)^2 + 2000^2 + (-2000)^2} = 3464.101615 \text{ m}$$

$$|\bar{r} - \bar{r}_2| = |(2000 + 6000)\hat{a}_x - 2000\hat{a}_y + (1000 + 3000)\hat{a}_z| = |8000\hat{a}_x - 2000\hat{a}_y + 4000\hat{a}_z|$$
$$= \sqrt{8000^2 + (-2000)^2 + 4000^2} = 9165.15139 \text{ m}$$

Plugging these into (4.66),

$$V_S = \frac{15\times10^{-6}}{4\pi(8.8541878\times10^{-12})3464.1} + \frac{-20\times10^{-6}}{4\pi(8.8541878\times10^{-12})9165.15}$$
$$= 38.91724 - 19.6124459$$
$$= 19.30479 \text{ V}$$

$V_S =$ **19.305 V**

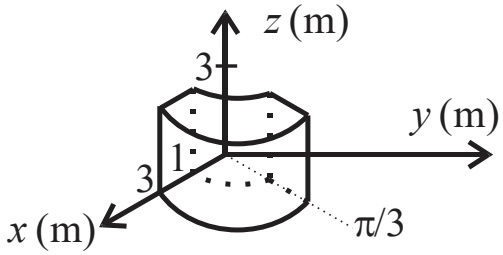
2) A ghastly vector field $\bar{G} = \frac{2}{\rho}\hat{a}_\rho - 6\rho^2 z\,\hat{a}_\phi + 4z\cos\phi\,\hat{a}_z$ (ghosts/m^2) exists at Haunted Castle, find the divergence and curl of $\bar{G}$. Then, find the outward flux Ψ_G of $\bar{G}$ through the closed surface described by $1 \le \rho \le 3$ m, $0 < \phi < \pi/3$, and $0 \le z \le 3$ m.

$$\text{div } \bar{G} = \bar{\nabla}\bullet\bar{G} = \frac{1}{\rho}\frac{\partial(\rho G_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial G_\phi}{\partial\phi} + \frac{\partial G_z}{\partial z} = \frac{1}{\rho}\frac{\partial(2)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(-6\rho^2 z)}{\partial\phi} + \frac{\partial(4z\cos\phi)}{\partial z}$$
$$= 0 + 0 + 4\cos\phi = 4\cos\phi \text{ (ghosts/m}^3\text{)}$$

$$\text{curl } \bar{G} = \bar{\nabla}\times\bar{G} = \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial G_z}{\partial\phi} - \frac{\partial G_\phi}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial\rho}\right) + \hat{a}_z\left(\frac{1}{\rho}\frac{\partial(\rho G_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial G_\rho}{\partial\phi}\right)$$
$$= \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial(4z\cos\phi)}{\partial\phi} - \frac{\partial(-6\rho^2 z)}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial(2/\rho)}{\partial z} - \frac{\partial(4z\cos\phi)}{\partial\rho}\right)$$
$$+ \hat{a}_z\left(\frac{1}{\rho}\frac{\partial(-6\rho^3 z)}{\partial\rho} - \frac{1}{\rho}\frac{\partial(2/\rho)}{\partial\phi}\right)$$
$$= \hat{a}_\rho\left(\frac{-4z\sin\phi}{\rho} + 6\rho^2\right) + \hat{a}_\phi(0-0) + \hat{a}_z\left(\frac{-18\rho^2 z}{\rho} - 0\right) \text{ (ghosts/m}^3\text{)}$$

Use divergence theorem to get the outward flux Ψ_G of $\bar{G}$ through the closed surface described by $1 \le \rho \le 3$ m, $0 < \phi < \pi/3$, and $0 \le z \le 3$ m.

$$\Psi_G = \oiint_S \bar{G}\bullet d\bar{s} = \iiint_V \bar{\nabla}\bullet\bar{G}\,dv = \iiint_V 4\cos\phi\;\rho\,d\rho\,d\phi\,dz$$
$$= 4\int_{z=0}^{3} dz \int_{\phi=0}^{\pi/3}\cos\phi\,d\phi \int_{\rho=1}^{3}\rho\,d\rho$$
$$= 4(3-0)(\sin\phi)\Big|_{\phi=0}^{\pi/3}\left(\frac{\rho^2}{2}\right)\Big|_{\rho=1}^{3}$$
$$= 12(\sin\pi/3 - 0)(9/2 - 1/2) = 41.5692 \text{ (ghosts)}$$



curl $\bar{G}$ = $\bar{\nabla}\times\bar{G} = \left(\frac{-4z\sin\phi}{\rho} + 6\rho^2\right)\hat{a}_\rho - 18\rho z\,\hat{a}_z$ **(ghosts/m^3)**

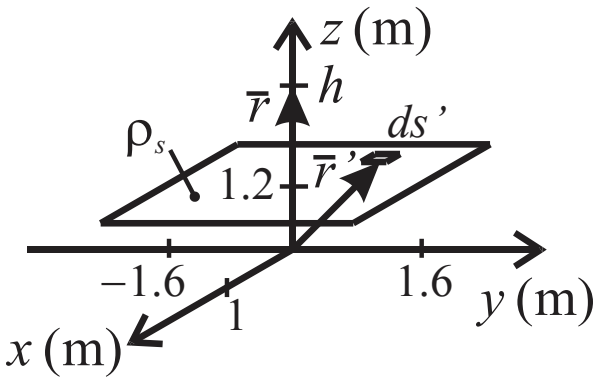
div $\bar{G}$ = **4 cos ϕ (ghosts/m^3)** Ψ_G = **41.569 (ghosts)**

Extra Credit: Is $\bar{G}$: divergenceless? Y / **N**; irrotational? Y / **N**; conservative? Y / **N**

3) Using the power of lightning, Dr. Frankenstein has placed a uniform surface charge density -24 μC/m^2 on his laboratory slab, a rectangular surface ($-1 \le x \le 1$ m, $-1.6 \le y \le 1.6$ m, & $z = 1.2$ m) suspended in free space. Find indicated quantities needed to calculate the electric field at a point (0, 0, h) along the z-axis above the slab. (Do **NOT** attempt to solve, simplify, or perform any coordinate transformations.)

a) Circle most appropriate coordinate system for problem- Cartesian

b) Draw a fully-labeled sketch of the problem geometry, include differentials, position vectors ...



c) Appropriate general integral to find $\bar{E}$ in position vector form (do NOT define terms or set limits) is:

$$\bar{E} = \iint_{s'} \frac{\rho_s (\bar{r} - \bar{r}')\, ds'}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}'\right|^3}$$

d) Determine specific **field** position vector:

At point (0, 0, h), $\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z = h\,\hat{a}_z$

$$\bar{r} = h\,\hat{a}_z$$

e) Determine specific **source** position vector:

On surface for ρ_s, $\bar{r}' = x'\,\hat{a}_x + y'\,\hat{a}_y + z'\,\hat{a}_z = x'\,\hat{a}_x + y'\,\hat{a}_y + 1.2\,\hat{a}_z$ (m)

$$\bar{r}' = x'\,\hat{a}_x + y'\,\hat{a}_y + 1.2\,\hat{a}_z \text{ (m)}$$

f) Determine **source** differential, i.e., circle correct one, and write-out specific expression

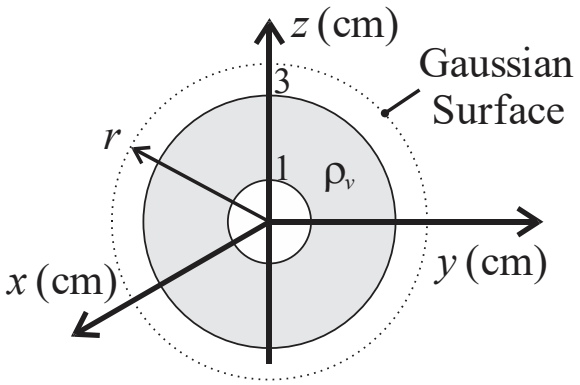
$$ds' = |d\bar{s}_z'| = dx'dy'$$

g) Use above to set-up the specific integral for $\bar{E}$, including limits. Do **NOT** simplify or solve.

$$\bar{E} = \int_{y'=-1.6}^{1.6} \int_{x'=-1}^{1} \frac{-24\times10^{-6}\,[h\,\hat{a}_z - (x'\,\hat{a}_x + y'\,\hat{a}_y + 1.2\,\hat{a}_z)]\,dx'dy'}{4\pi(8.854\times10^{-12})\left|h\,\hat{a}_z - (x'\,\hat{a}_x + y'\,\hat{a}_y + 1.2\,\hat{a}_z)\right|^3} \text{ (V/m)}$$

4) To combat Aragorn and Baggins, Sauron creates a plasma ball which manifests as a uniform volume charge density $\rho_v = 8\ \mu C/m^3$ constrained in a spherical shell $1 \le r \le 3$ cm suspended in free space at the origin. Find the total charge Q_S contained in the plasma ball. Then, find the electric flux density vectors at spherical points A(0.8 cm, π/2, 3π/2) & B(3.8 cm, π/3, 4π/3). (Hint: Gauss' Law)

$$Q_S = \iiint_V \rho_v\, dV = \iiint_V 8\times10^{-6}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$
$$= 8\times10^{-6} \int_{r=0.01}^{0.03} r^2\, dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$
$$= 8\times10^{-6} \left(\frac{r^3}{3}\right)\Bigg|_{r=0.01}^{0.03} (-\cos\theta)\Big|_{\theta=0}^{\pi} (\phi)\Big|_{\phi=0}^{2\pi}$$
$$= 8\times10^{-6}(0.03^3/3 - 0.01^3/3)(1+1)(2\pi - 0)$$
$$= 8.7126836\times10^{-10}\ \text{(C)}$$



Per Gauss' Law $\psi = \oiint_s \bar{D} \bullet d\bar{s} = Q_{enc}$. By spherical symmetry, we expect the electric flux to be in the radial direction, $\bar{D} = \hat{a}_r D_r$ and constant at a given radius r. Choose a sphere of radius r as our Gaussian surface. Then,

$$\oiint_s \bar{D} \bullet d\bar{s} = \oiint_s \hat{a}_r D_r \bullet d\bar{s}_r = D_r \oiint_s ds_r = D_r 4\pi r^2.$$

At point A(0.8 cm, π/2, 3π/2), we are **inside** the plasma ball. Therefore, $Q_{enc} = 0$ and we get

$$\oiint_s \bar{D} \bullet d\bar{s} = D_{r,A} 4\pi (0.008)^2 = Q_{enc,A} = 0 \Rightarrow \underline{\boldsymbol{D_{r,A} = 0}}.$$

At point B(3.8 cm, π/3, 4π/3), we are **outside** the plasma ball. Therefore, $Q_{enc} = Q_S$ and we get

$$\oiint_s \bar{D} \bullet d\bar{s} = D_{r,B} 4\pi (0.038)^2 = Q_{enc,B} = 8.7126836\times10^{-10} \Rightarrow \underline{\boldsymbol{D_{r,B} = 4.801488\times10^{-8}\ (C/m^2)}}.$$

$Q_S =$ **0.87127 (nC) = 871.27 (pC)** $\bar{D}_A =$ **0** $\bar{D}_B =$ $\hat{a}_r$ 48.015 **(nC/m²)**