

EE 381 Electric & Magnetic Fields Examination #1 (Fall 2026)Name KEY B

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheet in exam.

- 1) Given points $A(-2, 7, 8)$, $B(3, 1, -4)$, $C(6, 2, 5)$, & $D(6, -1, 9)$ [units of meters] and vectors $\vec{L} = 60\hat{a}_x - 23\hat{a}_y + 17\hat{a}_z$, $\vec{M} = 12\hat{a}_x + 33\hat{a}_y - 47\hat{a}_z$, and $\vec{N} = 45\hat{a}_x + 30\hat{a}_y + 50\hat{a}_z$, find:

- a) the **distance** from point D to B , and

Position vector to point D - $\vec{r}_D = 6\hat{a}_x - 1\hat{a}_y + 9\hat{a}_z$ (m)

Position vector to point B - $\vec{r}_B = 3\hat{a}_x + 1\hat{a}_y - 4\hat{a}_z$ (m)

Distance vector from point D to B -

$$\begin{aligned}\vec{r}_{DB} &= \vec{r}_B - \vec{r}_D = [3 - 6]\hat{a}_x + [1 - (-1)]\hat{a}_y + (-4 - 9)\hat{a}_z \\ &= -3\hat{a}_x + 2\hat{a}_y - 13\hat{a}_z \text{ (m)}\end{aligned}$$

Distance from point D to B -

$$d_{DB} = |\vec{r}_{DB}| = \sqrt{\vec{r}_{DB} \cdot \vec{r}_{DB}} = \sqrt{(-3)^2 + 2^2 + (-13)^2} = \sqrt{182} = 13.49074 \text{ (m)}$$

$$\underline{\underline{d_{DB} = 13.4907 \text{ m}}}$$

- b) the **unit vector** in the direction of $\vec{N}$,

$$\hat{a}_N = \frac{\vec{N}}{|\vec{N}|} = \frac{45\hat{a}_x + 30\hat{a}_y + 50\hat{a}_z}{\sqrt{45^2 + 30^2 + 50^2}} = \frac{45\hat{a}_x + 30\hat{a}_y + 50\hat{a}_z}{\sqrt{5425}}$$

$$\underline{\underline{\hat{a}_N = 0.61096\hat{a}_x + 0.40731\hat{a}_y + 0.67884\hat{a}_z}}$$

- c) the **scalar** and **vector** components of $\vec{L}$ in the direction of $\vec{N}$.

$$\begin{aligned}L_N &= \vec{L} \cdot \hat{a}_N = (60\hat{a}_x - 23\hat{a}_y + 17\hat{a}_z) \cdot (0.611\hat{a}_x + 0.407\hat{a}_y + 0.679\hat{a}_z) \\ &= 60(0.611) - 23(0.407) + 17(0.679) = 38.82982\end{aligned}$$

$$\begin{aligned}\vec{L}_N &= L_N \hat{a}_N = 38.83(0.611\hat{a}_x + 0.407\hat{a}_y + 0.679\hat{a}_z) \\ &= 23.7235\hat{a}_x + 15.81577\hat{a}_y + 26.3592\hat{a}_z\end{aligned}$$

scalar: $\underline{\underline{L_N = 38.83}}$

vector: $\underline{\underline{\vec{L}_N = 23.723\hat{a}_x + 15.816\hat{a}_y + 26.359\hat{a}_z}}$

- 2) Given points $A_{\text{cart}}(-3, 1, 4)$, $B_{\text{cyl}}(2, 41^\circ, -3)$, & $C_{\text{sph}}(9, 150^\circ, 30^\circ)$ [units of distance are meters] and vectors $\bar{L} = 8\hat{a}_x$, $\bar{M} = 3\hat{a}_y$, $\bar{N} = -4(x^2 + y^2)\hat{a}_z$, $\bar{P} = 6(x^2 + y^2)\hat{a}_z$, determine:

- a) the location of point C in **Cartesian** coordinates

$$x = r \sin \theta \cos \phi = 9 \sin 150^\circ \cos 30^\circ = 3.8971 \text{ (m)}$$

$$y = r \sin \theta \sin \phi = 9 \sin 150^\circ \sin 30^\circ = 2.25 \text{ (m)}$$

$$z = r \cos \theta = 9 \cos 150^\circ = -7.79423 \text{ (m)}$$

$$\underline{C_{\text{Cart}}(3.897, 2.25, -7.794) \text{ (m)}}$$

- b) an expression for the vector $\bar{P}$ in **cylindrical** and **spherical** coordinates, and

$$\text{Using } \rho = \sqrt{x^2 + y^2}, \text{ we get } \bar{P}_{\text{Cart}} = 6(x^2 + y^2)\hat{a}_z \Rightarrow \bar{P}_{\text{cyl}} = 6\rho^2\hat{a}_z$$

$$\text{Using } \rho = r \sin \theta \text{ and } \hat{a}_z = \cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta, \text{ we get}$$

$$\bar{P}_{\text{cyl}} = 6(r \sin \theta)^2 (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta) \Rightarrow \bar{P}_{\text{sph}} = 6r^2 \sin^2 \theta (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta)$$

$$\bar{P}_{\text{cyl}} = \underline{6\rho^2\hat{a}_z}$$

$$\bar{P}_{\text{sph}} = \underline{6r^2 \sin^2 \theta (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta)}$$

- c) the vector $\bar{M}$ in **cylindrical** coordinates evaluated at point B_{cyl} .

$$\text{Using } \hat{a}_y = \sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi, \bar{M}_{\text{cyl}} = 3(\sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi).$$

$$\text{At point } B, \bar{M}_{\text{cyl}} = 3(\sin 41^\circ \hat{a}_\rho + \cos 41^\circ \hat{a}_\phi) = 1.968\hat{a}_\rho + 2.264\hat{a}_\phi$$

$$\bar{M}_{\text{cyl},B} = \underline{1.968\hat{a}_\rho + 2.264\hat{a}_\phi}$$

- 3) A Belgian coaxial transmission line is made with brass conductors ($\sigma_b = 1.1 \times 10^7 \text{ S/m}$), a center wire of diameter 3 mm and shield of diameter 9 mm, separated by cocoa powder ($\epsilon_{\text{cocoa}} = 2.1\epsilon_0$, $\sigma_{\text{cocoa}} = 2 \times 10^{-7} \text{ S/m}$). It is operated at 210 MHz. Find the skin depth δ as well as the resistance R , inductance L , conductance G , and capacitance C per-unit-length. Next, find the attenuation constant α (Np/m & dB/m) and phase constant β (rad/m).

Conductors are brass - $\epsilon_b = \epsilon_0$, $\sigma_b = 1.1 \times 10^7 \text{ S/m}$, $\mu_b = \mu_0$ (non-magnetic)

Cocoa powder insulation- $\epsilon_{\text{cocoa}} = 2.1\epsilon_0$, $\sigma_{\text{cocoa}} = 2 \times 10^{-7} \text{ S/m}$, $\mu_{\text{cocoa}} = \mu_0$ (non-magnetic)

Per Table 11.1

$$\delta = \frac{1}{\sqrt{\pi f \mu_b \sigma_b}} = \frac{1}{\sqrt{\pi (210 \cdot 10^6) 4\pi \cdot 10^{-7} (1.1 \cdot 10^7)}} \Rightarrow \underline{\delta = 10.47163 \times 10^{-6} \text{ m}}$$

$$R = \frac{1}{2\pi \delta \sigma_b} \left(\frac{1}{a} + \frac{1}{b} \right) = \frac{1}{2\pi (10.472 \cdot 10^{-6}) 1.1 \cdot 10^7} \left(\frac{1}{1.5 \cdot 10^{-3}} + \frac{1}{4.5 \cdot 10^{-3}} \right) \Rightarrow \underline{R = 1.2282 \Omega/\text{m}}$$

$$L = \frac{\mu_{\text{cocoa}}}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{4\pi \cdot 10^{-6}}{2\pi} \ln\left(\frac{4.5 \cdot 10^{-3}}{1.5 \cdot 10^{-3}}\right) \Rightarrow \underline{L = 2.19722 \times 10^{-7} \text{ H/m}}$$

H/m

$$G = \frac{2\pi \sigma_{\text{cocoa}}}{\ln(b/a)} = \frac{2\pi \cdot 2 \cdot 10^{-7}}{\ln(4.5/1.5)} \Rightarrow \underline{G = 1.14384 \times 10^{-6} \text{ S/m}}$$

$$C = \frac{2\pi \epsilon_{\text{cocoa}}}{\ln(b/a)} = \frac{2\pi (2.1) 8.8541878 \cdot 10^{-12}}{\ln(4.5/1.5)} \Rightarrow \underline{C = 106.342 \times 10^{-12} \text{ F/m}}$$

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(1.2282 + j2\pi(210 \cdot 10^6) 2.197 \cdot 10^{-7})(1.144 \cdot 10^{-6} + j2\pi(210 \cdot 10^6) 106.34 \cdot 10^{-12})} \\ &\Rightarrow \underline{\gamma = 0.013536 + j 6.378067 \text{ 1/m}} \end{aligned}$$

$$\text{Attenuation constant } \alpha = \text{Re}(\gamma) = \text{Re}(0.013536 + j 6.378067) \Rightarrow \underline{\alpha = 0.013536 \text{ Np/m}}$$

$$\text{In dB/m, } \alpha * [20 \log(e)] = 0.013536 * 8.68589 \Rightarrow \underline{\alpha = 0.117569 \text{ dB/m}}$$

$$\text{Phase constant } \beta = \text{Im}(\gamma) = \text{Im}(0.013536 + j 6.378067) \Rightarrow \underline{\beta = 6.378067 \text{ Np/m}}$$

$$\delta = \underline{10.4716 \mu\text{m}} \quad R = \underline{1.2282 \Omega/\text{m}} \quad L = \underline{219.722 \text{ nH/m}}$$

$$G = \underline{1.1438 \mu\text{S/m}} \quad C = \underline{106.342 \text{ pF/m}}$$

$$\alpha = \underline{0.013536 \text{ Np/m}} = \underline{0.11757 \text{ dB/m}}$$

$$\beta = \underline{6.378067 \text{ Np/m}}$$

- 4) For a lossless transmission circuit, $Z_0 = 50 \Omega$, $u = 1.8 \times 10^8 \text{ m/s}$, $\ell = 1.7346\lambda$, $f = 320 \text{ MHz}$, $Z_g = 50 \Omega$, and $V_g = 30 \angle 0^\circ \text{ (V)}$. If an input reflection coefficient of $0.632 \angle 71.9^\circ$ is measured, **use a Smith chart** to find the input impedance, load reflection coefficient, load impedance, and standing wave ratio. Show and label all work on Smith chart for credit. Analytic answers are not accepted.

Wavelength $\lambda = u / f = 1.8 \times 10^8 / 320 \times 10^6 \Rightarrow \underline{\lambda = 0.5625 \text{ m} = 56.25 \text{ cm}}$.

Steps

- 1) Plot Γ_{in} . Set compass to radius corresponding to $|\Gamma| = 0.632$ using scale at bottom of Smith chart. Draw $|\Gamma| = 0.632$ circle, centered on Smith chart. Draw radial line from center of Smith chart through $\angle \Gamma_{in} = 71.9^\circ$ on “ANGLE OF REFLECTION COEFFICIENT” scale.
- 2) Also, draw arcs on the “REFL. COEFF, V or I” and “SWR (VSWR)” scales at bottom of Smith chart. Read $|\Gamma| = 0.632$ and **SWR = 4.42**.
- 3) The z_{in} / Γ_{in} point is at the intersection of the circle and radial line. Using r circles and x arcs, read **$z_{in} = 0.6 + j1.2 \Omega/\Omega$** .
- 4) Compute input impedance $Z_{in} = Z_0 z_{in} = 50 (0.6 + j 1.2) \Rightarrow \underline{Z_{in} = 30 + j 60 \Omega}$.
- 5) To get to the load point, we must go $\ell/\lambda = 1.7346 \Rightarrow 0.2346 = 0.1501 + 0.0845$ in “WAVELENGTHS TOWARD LOAD” direction along $|\Gamma| = 0.632$ circle from z_{in} / Γ_{in} point to load point. Draw radial line from center of Smith chart through z_L / Γ_L point.
- 6) Read $\angle \Gamma_L = -119^\circ$ using “ANGLE OF REFLECTION COEFFICIENT” scale. We already know that $|\Gamma| = 0.632$ on a lossless transmission line. $\Rightarrow \underline{\Gamma_L = 0.632 \angle -119^\circ}$.
- 7) Using r circles and x arcs, read the normalized load impedance **$z_L = 0.3 - j 0.55 \Omega/\Omega$** .
- 8) Compute the load impedance $Z_L = Z_0 z_{in} = 50 (0.3 - j 0.55) \Rightarrow \underline{Z_L = 15 - j 27.5 \Omega}$.

input impedance = **$Z_{in} = 30 + j 60 \Omega$** load refl. coeff. = **$\Gamma_L = 0.632 \angle -119^\circ$**

load impedance = **$Z_L = 15 - j 27.5 \Omega$** SWR = **4.42**

Express reflection coefficients in $A \angle \theta$ (deg) form & impedance/admittance in rectangular form.

Simple Smith Chart

$Z_0 = 50 \Omega$
 $\lambda = 56.25 \text{ cm}$

