

EE 381 Electric & Magnetic Fields Examination #1 (Fall 2026)Name KEY A

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheet in exam.

- 1) Given points $A(-2, 7, 8)$, $B(3, 1, -4)$, $C(6, 2, 5)$, and $D(6, -1, 9)$ [units of meters], and vectors $\vec{L} = 60\hat{a}_x - 10\hat{a}_y + 15\hat{a}_z$, $\vec{M} = -12\hat{a}_x + 10\hat{a}_y - 47\hat{a}_z$, and $\vec{N} = 45\hat{a}_x + 30\hat{a}_y + 52\hat{a}_z$ find:

- a) the **distance** from point A to point C , and

Position vector to point A - $\vec{r}_A = -2\hat{a}_x + 7\hat{a}_y + 8\hat{a}_z$ (m)

Position vector to point C - $\vec{r}_C = 6\hat{a}_x + 2\hat{a}_y + 5\hat{a}_z$ (m)

Distance vector from point C to A -

$$\begin{aligned}\vec{r}_{CA} &= \vec{r}_C - \vec{r}_A = [6 - (-2)]\hat{a}_x + (2 - 7)\hat{a}_y + (5 - 8)\hat{a}_z \\ &= 8\hat{a}_x - 5\hat{a}_y - 3\hat{a}_z \text{ (m)}\end{aligned}$$

Distance from point C to A -

$$d_{CA} = |\vec{r}_{CA}| = \sqrt{\vec{r}_{CA} \cdot \vec{r}_{CA}} = \sqrt{8^2 + (-5)^2 + (-3)^2} = \sqrt{98} = 9.8995 \text{ (m)}$$

$$\underline{d_{CA} = 9.8995 \text{ m}}$$

- b) the **unit vector** in the direction of $\vec{L}$,

$$\hat{a}_L = \frac{\vec{L}}{|\vec{L}|} = \frac{60\hat{a}_x - 10\hat{a}_y + 15\hat{a}_z}{\sqrt{60^2 + (-10)^2 + 15^2}} = \frac{60\hat{a}_x - 10\hat{a}_y + 15\hat{a}_z}{\sqrt{3925}}$$

$$\underline{\hat{a}_L = 0.9577\hat{a}_x - 0.1596\hat{a}_y + 0.2394\hat{a}_z}$$

- c) the **scalar** and **vector** components of $\vec{M}$ in the direction of $\vec{L}$.

$$\begin{aligned}M_L &= \vec{M} \cdot \hat{a}_L = (-12\hat{a}_x + 10\hat{a}_y - 47\hat{a}_z) \cdot (0.9577\hat{a}_x - 0.1596\hat{a}_y + 0.2394\hat{a}_z) \\ &= (-12)(0.9577) + 10(-0.1596) - 47(0.2394) = -24.34165\end{aligned}$$

$$\begin{aligned}\vec{M}_L &= M_L \hat{a}_L = -24.34165(0.9577\hat{a}_x - 0.1596\hat{a}_y + 0.2394\hat{a}_z) \\ &= -23.312\hat{a}_x + 3.885\hat{a}_y - 5.828\hat{a}_z\end{aligned}$$

scalar: $\underline{M_L = -24.342}$

vector: $\underline{\vec{M}_L = -23.312\hat{a}_x + 3.885\hat{a}_y - 5.828\hat{a}_z}$

- 2) Given points $A_{\text{cart}}(3, -1, 4)$, $B_{\text{cyl}}(2, 71^\circ, -5)$, & $C_{\text{sph}}(4, 50^\circ, 250^\circ)$ [units of distance are meters] and vectors $\bar{L} = 3\hat{a}_y$, $\bar{M} = 2\hat{a}_x$, $\bar{N} = -4(x^2 + y^2)\hat{a}_z$, $\bar{P} = 6(x^2 + y^2)\hat{a}_y$, determine:

- a) the location of point C in **Cartesian** coordinates,

$$x = r \sin \theta \cos \phi = 4 \sin 50^\circ \cos 250^\circ = -1.048 \text{ (m)}$$

$$y = r \sin \theta \sin \phi = 4 \sin 50^\circ \sin 250^\circ = -2.8794 \text{ (m)}$$

$$z = r \cos \theta = 4 \cos 50^\circ = 2.57115 \text{ (m)}$$

$$\underline{C_{\text{Cart}}(-1.048, -2.879, 2.571) \text{ (m)}}$$

- b) an expression for the vector $\bar{N}$ in **cylindrical** and **spherical** coordinates, and

$$\text{Using } \rho = \sqrt{x^2 + y^2}, \text{ we get } \bar{N}_{\text{Cart}} = -4(x^2 + y^2)\hat{a}_z \Rightarrow \bar{N}_{\text{cyl}} = -4\rho^2 \hat{a}_z$$

$$\text{Using } \rho = r \sin \theta \text{ and } \hat{a}_z = \cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta, \text{ we get}$$

$$\bar{N}_{\text{cyl}} = -4(r \sin \theta)^2 (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta) \Rightarrow \bar{N}_{\text{sph}} = -4r^2 \sin^2 \theta (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta)$$

$$\bar{N}_{\text{cyl}} = \underline{-4\rho^2 \hat{a}_z}$$

$$\bar{N}_{\text{sph}} = \underline{-4r^2 \sin^2 \theta (\cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta)}$$

- c) the vector $\bar{L}$ in **cylindrical** coordinates evaluated at point B_{cyl} .

$$\text{Using } \hat{a}_y = \sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi, \bar{L}_{\text{cyl}} = 3(\sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi).$$

$$\text{At point B, } \bar{L}_{\text{cyl}} = 3(\sin 71^\circ \hat{a}_\rho + \cos 71^\circ \hat{a}_\phi) = 2.8366 \hat{a}_\rho + 0.9767 \hat{a}_\phi$$

$$\bar{L}_{\text{cyl},B} = \underline{2.8366 \hat{a}_\rho + 0.9767 \hat{a}_\phi}$$

- 3) An Albanian coaxial transmission line is made with brass conductors ($\sigma_b = 10^7$ S/m), a center wire of diameter 2 mm and shield of diameter 10 mm, separated by beeswax ($\epsilon_{bw} = 2.2\epsilon_0$, $\sigma_{bw} = 10^{-7}$ S/m). It is operated at 220 MHz. Find the skin depth δ as well as the resistance R , inductance L , conductance G , and capacitance C per-unit-length. Next, find the attenuation constant α (Np/m & dB/m) and phase constant β (rad/m).

Conductors are brass - $\epsilon_b = \epsilon_0$, $\sigma_b = 10^7$ S/m, $\mu_b = \mu_0$ (non-magnetic)

Insulation is beeswax - $\epsilon_{bw} = 2.2\epsilon_0$, $\sigma_{bw} = 10^{-7}$ S/m, $\mu_{bw} = \mu_0$ (non-magnetic)

Per Table 11.1

$$\delta = \frac{1}{\sqrt{\pi f \mu_b \sigma_b}} = \frac{1}{\sqrt{\pi(220 \cdot 10^6)4\pi \cdot 10^{-7}(10^7)}} \Rightarrow \underline{\delta = 10.73022 \times 10^{-6} \text{ m}}$$

$$R = \frac{1}{2\pi\delta\sigma_b} \left(\frac{1}{a} + \frac{1}{b} \right) = \frac{1}{2\pi(10.73022 \cdot 10^{-6})10^7} \left(\frac{1}{10^{-3}} + \frac{1}{5 \cdot 10^{-3}} \right) \Rightarrow \underline{R = 1.77989 \Omega/\text{m}}$$

$$L = \frac{\mu_{bw}}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{4\pi \cdot 10^{-6}}{2\pi} \ln\left(\frac{5 \cdot 10^{-3}}{10^{-3}}\right) \Rightarrow \underline{L = 3.21888 \times 10^{-7} \text{ H/m}}$$

$$G = \frac{2\pi\sigma_{bw}}{\ln(b/a)} = \frac{2\pi \cdot 10^{-7}}{\ln(5/1)} \Rightarrow \underline{G = 3.9039625 \times 10^{-7} \text{ S/m}}$$

$$C = \frac{2\pi\epsilon_{bw}}{\ln(b/a)} = \frac{2\pi(2.2)8.8541878 \cdot 10^{-12}}{\ln(5/1)} \Rightarrow \underline{C = 76.046 \times 10^{-12} \text{ F/m}}$$

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(1.7799 + j2\pi(220 \cdot 10^6)3.219 \cdot 10^{-7})(3.904 \cdot 10^{-7} + j2\pi(220 \cdot 10^6)76.05 \cdot 10^{-12})} \\ &\Rightarrow \underline{\gamma = 0.013692 + j 6.839023 \text{ 1/m}} \end{aligned}$$

$$\text{Attenuation constant } \alpha = \text{Re}(\gamma) = \text{Re}(0.013692 + j 6.839023) \Rightarrow \underline{\alpha = 0.013692 \text{ Np/m}}$$

$$\text{In dB/m, } \alpha^*[20 \log(e)] = 0.013692 * 8.68589 \Rightarrow \underline{\alpha = 0.118923 \text{ dB/m}}$$

$$\text{Phase constant } \beta = \text{Im}(\gamma) = \text{Im}(0.013692 + j 6.839023) \Rightarrow \underline{\beta = 6.839023 \text{ Np/m}}$$

$$\delta = \underline{10.7302 \mu\text{m}} \quad R = \underline{1.7799 \Omega/\text{m}} \quad L = \underline{321.888 \text{ nH/m}}$$

$$G = \underline{0.3904 \mu\text{S/m}} = \underline{390.396 \text{ nS/m}} \quad C = \underline{76.046 \text{ pF/m}}$$

$$\alpha = \underline{0.013692 \text{ Np/m}} = \underline{0.11892 \text{ dB/m}} \quad \beta = \underline{6.839023 \text{ Np/m}}$$

- 4) For a lossless transmission circuit, $Z_0 = 75 \Omega$, $u = 1.6 \times 10^8$ m/s, $\ell = 1.7493\lambda$, $f = 330$ MHz, $Z_g = 75 \Omega$, and $V_g = 20 \angle 0^\circ$ (V). If an input reflection coefficient of $0.62 \angle 97.3^\circ$ is measured, **use a Smith chart** to find the input impedance, load reflection coefficient, load impedance, and standing wave ratio. Show and label all work on Smith chart for credit. Analytic answers are not accepted.

Wavelength $\lambda = u / f = 1.6 \times 10^8 / 330 \times 10^6 \Rightarrow \lambda = 0.48485 \text{ m} = 48.485 \text{ cm}.$

Steps

- 1) Plot Γ_{in} . Set compass to radius corresponding to $|\Gamma| = 0.62$ using scale at bottom of Smith chart. Draw $|\Gamma| = 0.62$ circle, centered on Smith chart. Draw radial line from center of Smith chart through $\angle \Gamma_{in} = 97.3^\circ$ on “ANGLE OF REFLECTION COEFFICIENT” scale.
- 2) Also, draw arcs on the “REFL. COEFF, V or I” and “SWR (VSWR)” scales at bottom of Smith chart. Read $|\Gamma| = 0.62$ and **SWR = 4.26**.
- 3) The z_{in} / Γ_{in} point is at the intersection of the circle and radial line. Using r circles and x arcs, read **$z_{in} = 0.4 + j0.8 \Omega/\Omega$** .
- 4) Compute input impedance $Z_{in} = Z_0 z_{in} = 75 (0.4 + j 0.8) \Rightarrow \mathbf{Z_{in} = 30 + j 60 \Omega}.$
- 5) To get to the load point, we must go $\ell/\lambda = 1.7493 \Rightarrow 0.2493 = 0.1148 + 0.1345$ in “WAVELENGTHS TOWARD LOAD” direction along the $|\Gamma| = 0.62$ circle from z_{in} / Γ_{in} point to load point. Draw radial line from center of Smith chart through z_L / Γ_L point.
- 6) Read $\angle \Gamma_L = -83.1^\circ$ using “ANGLE OF REFLECTION COEFFICIENT” scale. We already know that $|\Gamma| = 0.62$ on a lossless transmission line. $\Rightarrow \mathbf{\Gamma_L = 0.62 \angle -83.1^\circ}.$
- 7) Using r circles and x arcs, read the normalized load impedance **$z_L = 0.5 - j 1 \Omega/\Omega$** .
- 8) Compute the load impedance $Z_L = Z_0 z_{in} = 75 (0.5 - j 1) \Rightarrow \mathbf{Z_L = 37.5 - j 75 \Omega}.$

input impedance = **$\mathbf{Z_{in} = 30 + j 60 \Omega}$** load refl. coeff. = **$\mathbf{\Gamma_L = 0.62 \angle -83.1^\circ}$**

load impedance = **$\mathbf{Z_L = 37.5 - j 75 \Omega}$** SWR = **$\mathbf{4.26}$**

Express reflection coefficients in $A \angle \theta$ (deg) form & impedance/admittance in rectangular form.

