EE 381 Electric and Magnetic Fields Quiz #6 (Fall 2025)

Name <u>**KEY A**</u>

Instructions: **Open book & notes.** Place answers in indicated spaces and show all work for full/partial credit.

For the vector field $\overline{T} = -3r\,\hat{a}_r + \dfrac{4}{\sin\theta}\hat{a}_\theta$ (tables/m), determine:

a) The divergence of $\overline{T}$.

$$\text{div } \overline{T} = \overline{\nabla}\bullet\overline{T} = \frac{1}{r^2}\frac{\partial\left(r^2 T_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta\, T_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial T_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial\left(-3r^3\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(4)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(0)}{\partial\phi}$$

$$= \frac{-3(3r^2)}{r^2} + 0 + 0 = -9$$

div $\overline{T}$ = $\overline{\nabla}\bullet\overline{T}$ = **-9 (tables/m^2)**

b) The curl of $\overline{T}$.

curl $\overline{T} = \overline{\nabla}\times\overline{T}$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial\left(\sin\theta\, T_\phi\right)}{\partial\theta} - \frac{\partial T_\theta}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial T_r}{\partial\phi} - \frac{1}{r}\frac{\partial\left(rT_\phi\right)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(rT_\theta\right)}{\partial r} - \frac{\partial T_r}{\partial\theta}\right]\right\}$$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial(0)}{\partial\theta} - \frac{\partial\left(\frac{4}{\sin\theta}\right)}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial(-3r)}{\partial\phi} - \frac{1}{r}\frac{\partial(0)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(\frac{4r}{\sin\theta}\right)}{\partial r} - \frac{\partial(-3r)}{\partial\theta}\right]\right\}$$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}[0-0]\right\} + \hat{a}_\theta[0-0] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{4}{\sin\theta} - 0\right]\right\} = \frac{4}{r\sin\theta}\hat{a}_\phi$$

curl $\overline{T}$ = $\overline{\nabla}\times\overline{T} = \dfrac{4}{r\sin\theta}\hat{a}_\phi$ (tables/m^2)

c) Is $\overline{T}$ solenoidal (or divergenceless)? Yes / $\boxed{\text{No}}$ (circle correct)

Why? $\overline{\nabla}\bullet\overline{T} = -9$ (tables/m^2) $\neq 0$

d) Is $\overline{T}$ irrotational (or potential)? Yes / $\boxed{\text{No}}$ (circle correct)

Why? $\overline{\nabla}\times\overline{T} = \dfrac{4}{r\sin\theta}\hat{a}_\phi$ (tables/m^2) $\neq 0$

e) Is $\overline{T}$ conservative? Yes / $\boxed{\text{No}}$ (circle correct)

Why? $\overline{\nabla}\times\overline{T} \neq 0$ implies by Stoke's theorem that $\oint_L \overline{T}\bullet d\overline{l} = \iint_s (\overline{\nabla}\times\overline{T})\bullet d\overline{s} \neq 0$

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Name KEY B

Instructions: **Closed book & notes.** Place answers in indicated spaces and show all work for full/partial credit.

For the vector field $\bar{M} = 2r\,\hat{a}_r - \dfrac{9}{\sin\theta}\hat{a}_\theta$ (mugs/m), determine if $\bar{M}$ is:

a) The divergence of $\bar{M}$.

$$\text{div } \bar{M} = \bar{\nabla}\bullet\bar{M} = \frac{1}{r^2}\frac{\partial\left(r^2 M_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta\, M_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial M_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial\left(2r^3\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(-9)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(0)}{\partial\phi}$$

$$= \frac{2(3r^2)}{r^2} + 0 + 0 = 6$$

div $\bar{M}$ = $\bar{\nabla}\bullet\bar{M}$ **= 6 (mugs/m²)**

b) The curl of $\bar{M}$.

curl $\bar{M} = \bar{\nabla}\times\bar{M}$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial\left(\sin\theta M_\phi\right)}{\partial\theta} - \frac{\partial M_\theta}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial M_r}{\partial\phi} - \frac{1}{r}\frac{\partial\left(rM_\phi\right)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(rM_\theta\right)}{\partial r} - \frac{\partial M_r}{\partial\theta}\right]\right\}$$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial(0)}{\partial\theta} - \frac{\partial\left(\frac{-9}{\sin\theta}\right)}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial(2r)}{\partial\phi} - \frac{1}{r}\frac{\partial(0)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(\frac{-9r}{\sin\theta}\right)}{\partial r} - \frac{\partial(2r)}{\partial\theta}\right]\right\}$$

$$= \hat{a}_r\left\{\frac{1}{r\sin\theta}[0-0]\right\} + \hat{a}_\theta[0-0] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{-9}{\sin\theta} - 0\right]\right\} = \frac{-9}{r\sin\theta}\hat{a}_\phi$$

curl $\bar{M}$ = $\bar{\nabla}\times\bar{M} = \dfrac{-9}{r\sin\theta}\hat{a}_\phi$ (mugs/m²)

c) Is $\bar{M}$ solenoidal (or divergenceless)? Yes / **No** (circle correct)

Why? $\bar{\nabla}\bullet\bar{M} = 6$ (mugs/m²) $\neq 0$

d) Is $\bar{M}$ irrotational (or potential)? Yes / **No** (circle correct)

Why? $\bar{\nabla}\times\bar{M} = \dfrac{-9}{r\sin\theta}\hat{a}_\phi$ (mugs/m²) $\neq 0$

e) Is $\bar{M}$ conservative? Yes / **No** (circle correct)

Why? $\bar{\nabla}\times\bar{M} \neq 0$ implies by Stoke's theorem that $\oint_L \bar{M}\bullet d\bar{l} = \oiint_s (\bar{\nabla}\times\bar{M})\bullet d\bar{s} \neq 0$