Wednesday, October 8, 2025 A

EE 381 Electric and Magnetic Fields Quiz #5 (Fall 2025)

Name Key A

Instructions: **Closed book & notes.** Place answers in indicated spaces and show all work for full/partial credit.

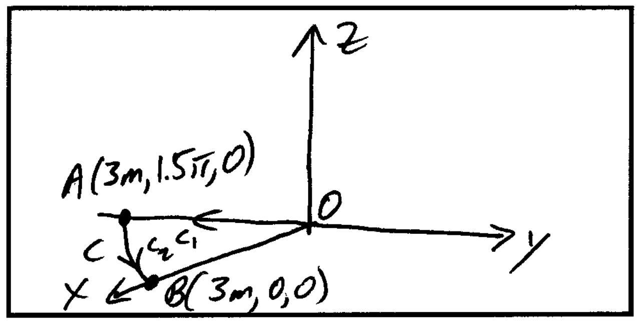
Useful equations: $d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

The path C goes radially from the origin to point A(3 m, 1.5π, 0) on the -y-axis and then on an arc of constant radius from point A to point B(3 m, 0, 0) on the +x-axis in the direction of increasing ϕ.

a) Sketch, with labels, the path C.

$1.5\pi \rightarrow 270°$ from $+x$-axis

$0 \rightarrow$ on $+x$-axis



b) For the vector field $\bar{L} = -7\rho\sin\phi\,\hat{a}_\rho + 2\rho\,\hat{a}_\phi + 6z\,\hat{a}_z$ (Lemurs/m), compute $\bar{L}\bullet d\bar{l}$.

$$\bar{L}\cdot d\bar{\ell} = (-7\rho\sin\phi\,\hat{a}_\rho + 2\rho\,\hat{a}_\phi + 6z\,\hat{a}_z)\cdot(d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z)$$

$\bar{L}\bullet d\bar{l} =$ $-7\rho\sin\phi\,d\rho + 2\rho^2\,d\phi + 6z\,dz$ (Lemurs)

c) Calculate the line integral $\int_C \bar{L}\bullet d\bar{l}$ $= \int_{0}^{A} \bar{L}\cdot d\bar{\ell} + \int_{A}^{B} \bar{L}\cdot d\bar{\ell}$

C_1 ($\phi = 1.5\pi$, $z = 0$) C_2 ($\rho = 3$m, $z = 0$)

Lemurs/m m

$$\int_C \bar{L}\cdot d\bar{\ell} = \int_{\rho=0}^{3} -7\rho\underbrace{\sin\phi}_{\sin 1.5\pi = -1}d\rho + 2\rho^2\underbrace{d\phi}_{0} + 6z\underbrace{dz}_{0}$$

$$+ \int_{\phi=1.5\pi}^{2\pi} -7\rho\sin\phi\underbrace{d\rho}_{0} + 2\underbrace{\rho^2}_{3^2}d\phi + 6z\underbrace{dz}_{0}$$

$$= +7\frac{\rho^2}{2}\Big|_{\rho=0}^{3} + 2(9)\,\phi\Big|_{1.5\pi}^{2\pi} = \frac{7}{2}(9-0) + 18(2\pi - 1.5\pi)$$

$\int_C \bar{L}\bullet d\bar{l} =$ 59.774 (Lemurs)

EE 381 Electric and Magnetic Fields Quiz #5 (Fall 2025)

Name Key B

Instructions: **Closed book & notes.** Place answers in indicated spaces and show all work for full/partial credit.

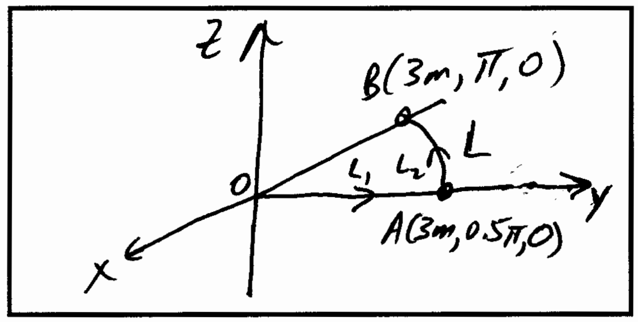
Useful equations: $d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z = \underline{d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

The path L goes radially from the origin to point $A(3\text{ m}, 0.5\pi, 0)$ on the $\underline{+y\text{-axis}}$ and then on an arc of constant radius from point A to point $B(3\text{ m}, \pi, 0)$ on the $\underline{-x\text{-axis}}$ in the direction of increasing ϕ.

a) Sketch, with labels, the path L.

$0.5\pi \rightarrow 90^\circ$ from $+x$-axis

$\pi \rightarrow 180^\circ$ from $+x$-axis



b) For the vector field $\bar{M} = 5\rho\sin\phi\,\hat{a}_\rho - 6\rho\,\hat{a}_\phi - 4z\,\hat{a}_z$ (Meerkats/m), compute $\bar{M}\bullet d\bar{l}$.

$$\bar{M}\cdot d\bar{l} = (5\rho\sin\phi\,\hat{a}_\rho - 6\rho\,\hat{a}_\phi - 4z\,\hat{a}_z)\cdot(d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z)$$

$\frac{\text{Meerkats}}{m}$ m

$\bar{M}\bullet d\bar{l} =$ $5\rho\sin\phi\,d\rho - 6\rho^2 d\phi - 4z\,dz$ (Meerkats)

c) Calculate the line integral $\int_L \bar{M}\bullet d\bar{l}$ $= \int_{0,\,L_1}^{A} \bar{M}\cdot d\bar{l} + \int_{A,\,L_2}^{B} \bar{M}\cdot d\bar{l}$

$\begin{pmatrix}\phi = 0.5\pi \\ z = 0\end{pmatrix}$ $\begin{pmatrix}\rho = 3m \\ z = 0\end{pmatrix}$

$\frac{\text{Meerkats}}{m}$ m

$$\int_L \bar{M}\cdot d\bar{l} = \int_{\rho=0}^{3m} 5\rho\underbrace{\sin\phi}_{\sin 0.5\pi = 1}\,d\rho - 6\rho^2\underbrace{d\phi}_{0} - 4z\underbrace{dz}_{0}$$

$$+ \int_{\phi=0.5\pi}^{\pi} 5\rho\sin\phi\,\underbrace{d\rho}_{0} - 6\underbrace{\rho^2}_{3^2}d\phi - 4z\underbrace{dz}_{0}$$

$$= 5\left(\frac{\rho^2}{2}\right)\Big|_{\rho=0}^{3} - 6(9)\,\phi\Big|_{0.5\pi}^{\pi} = \frac{5}{2}(9-0) - 54(\pi - 0.5\pi)$$

$\int_L \bar{M}\bullet d\bar{l} =$ -62.323 (Meerkats)