

**EE 381 Electric and Magnetic Fields Quiz #4 (Fall 2025)**Name KEY A

Instructions: Closed book &amp; notes. Place answers in indicated spaces and show all work for credit.

Useful equations:  $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$ ,  $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \hat{a}_n$ ,  $\vec{r}_{\text{position}} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z$ Given vectors  $\vec{M} = -12\hat{a}_x - 8\hat{a}_y + 16\hat{a}_z$ ,  $\vec{L} = 6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z$ , &  $\vec{N} = 6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z$  and the Cartesian points  $A(2, -8, 3)$ ,  $B(-3, 4, -3)$ , and  $C(7, 4, 5)$ , with units of meters, find:a) The unit vector in the direction of  $\vec{L}$ .

$$|\vec{L}| = \sqrt{\vec{L} \cdot \vec{L}} = \sqrt{(6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z) \cdot (6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z)} = \sqrt{6^2 + 9^2 + (-4)^2} = \sqrt{133} = 11.53256$$

$$\hat{a}_L = \frac{\vec{L}}{|\vec{L}|} = \frac{6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z}{\sqrt{133}} = 0.520\hat{a}_x + 0.780\hat{a}_y - 0.347\hat{a}_z$$

$$\hat{a}_L = \underline{0.520\hat{a}_x + 0.780\hat{a}_y - 0.347\hat{a}_z}$$

b) The smallest angle  $\theta$  (in degrees) that  $\vec{L}$  makes with the positive z-axis

Use scalar product-

$$\vec{L} \cdot \hat{a}_z = |\vec{L}|(1) \cos \theta$$

$$(6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z) \cdot \hat{a}_z = -4 = \sqrt{133} \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} \left( \frac{-4}{\sqrt{133}} \right) = 110.2944^\circ$$

$$\theta = \underline{110.2944^\circ}$$

c) The distance vector  $\vec{d}_{AB}$  going from point  $A$  to  $B$ position vectors-  $\vec{r}_A = 2\hat{a}_x - 8\hat{a}_y + 3\hat{a}_z$  (m) and  $\vec{r}_B = -3\hat{a}_x + 4\hat{a}_y - 3\hat{a}_z$  (m)

distance vector-

$$\begin{aligned} \vec{d}_{AB} &= \vec{r}_B - \vec{r}_A = (-3\hat{a}_x + 4\hat{a}_y - 3\hat{a}_z) - (2\hat{a}_x - 8\hat{a}_y + 3\hat{a}_z) = (-3-2)\hat{a}_x + (4+8)\hat{a}_y + (-3-3)\hat{a}_z \\ &= -5\hat{a}_x + 12\hat{a}_y - 6\hat{a}_z \text{ (m)} \end{aligned}$$

$$\vec{d}_{AB} = \underline{-5\hat{a}_x + 12\hat{a}_y - 6\hat{a}_z \text{ (m)}}$$

d) The vector product of  $\vec{L}$  with  $\vec{N}$ :

$$\begin{aligned} \vec{L} \times \vec{N} &= (6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z) \times (6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z) \\ &= [9(-9) - (-4)(-12)]\hat{a}_x + [-4(6) - 6(-9)]\hat{a}_y + [6(-12) - 9(6)]\hat{a}_z \\ &= -129\hat{a}_x + 30\hat{a}_y - 126\hat{a}_z \text{ (m)} \end{aligned}$$

$$\text{vector product of } \vec{L} \text{ with } \vec{N} = \underline{\vec{L} \times \vec{N} = -129\hat{a}_x + 30\hat{a}_y - 126\hat{a}_z}$$

**EE 381 Electric and Magnetic Fields Quiz #4 (Fall 2025)**Name KEY B

Instructions: Closed book &amp; notes. Place answers in indicated spaces and show all work for credit.

Useful equations:  $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$ ,  $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \hat{a}_n$ ,  $\vec{r}_{\text{position}} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z$ Given vectors  $\vec{M} = -12\hat{a}_x - 8\hat{a}_y + 16\hat{a}_z$ ,  $\vec{L} = 6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z$ , &  $\vec{N} = 6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z$  and the Cartesian points  $A(2, -8, 3)$ ,  $B(-3, 4, -3)$ , and  $C(7, 4, 5)$ , with units of meters, find:

- a) The unit vector in the direction of
- $\vec{N}$
- .

$$|\vec{N}| = \sqrt{\vec{N} \cdot \vec{N}} = \sqrt{(6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z) \cdot (6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z)} = \sqrt{6^2 + (-12)^2 + (-9)^2} = \sqrt{261} = 16.1555$$

$$\hat{a}_N = \frac{\vec{N}}{|\vec{N}|} = \frac{6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z}{\sqrt{261}} = 0.371\hat{a}_x - 0.743\hat{a}_y - 0.557\hat{a}_z$$

$$\hat{a}_N = \underline{0.371\hat{a}_x - 0.743\hat{a}_y - 0.557\hat{a}_z}$$

- b) The smallest angle
- $\theta$
- (in degrees) that
- $\vec{N}$
- makes with the positive z-axis

Use scalar product-

$$\vec{N} \cdot \hat{a}_z = |\vec{N}|(1) \cos \theta$$

$$(6\hat{a}_x - 12\hat{a}_y - 9\hat{a}_z) \cdot \hat{a}_z = -9 = \sqrt{261} \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} \left( \frac{-9}{\sqrt{261}} \right) = 123.8545^\circ$$

$$\theta = \underline{123.8545^\circ}$$

- c) The distance vector
- $\vec{d}_{AC}$
- going from point
- $A$
- to
- $C$

position vectors-  $\vec{r}_A = 2\hat{a}_x - 8\hat{a}_y + 3\hat{a}_z$  (m) and  $\vec{r}_C = 7\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$  (m)

distance vector-

$$\vec{d}_{AC} = \vec{r}_C - \vec{r}_A = (7\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z) - (2\hat{a}_x - 8\hat{a}_y + 3\hat{a}_z) = (7-2)\hat{a}_x + [4-(-8)]\hat{a}_y + (5-3)\hat{a}_z$$

$$= 5\hat{a}_x + 12\hat{a}_y + 2\hat{a}_z \text{ (m)}$$

$$\vec{d}_{AC} = \underline{5\hat{a}_x + 12\hat{a}_y + 2\hat{a}_z \text{ (m)}}$$

- d) The vector product of
- $\vec{L}$
- with
- $\vec{M}$
- :

$$\vec{L} \times \vec{M} = (6\hat{a}_x + 9\hat{a}_y - 4\hat{a}_z) \times (-12\hat{a}_x - 8\hat{a}_y + 16\hat{a}_z)$$

$$= [9(16) - (-4)(-8)]\hat{a}_x + [-4(-12) - 6(16)]\hat{a}_y + [6(-8) - 9(-12)]\hat{a}_z$$

$$= 112\hat{a}_x - 48\hat{a}_y + 60\hat{a}_z \text{ (m)}$$

$$\text{vector product of } \vec{L} \text{ with } \vec{M} = \underline{\vec{L} \times \vec{M} = 112\hat{a}_x - 48\hat{a}_y + 60\hat{a}_z}$$