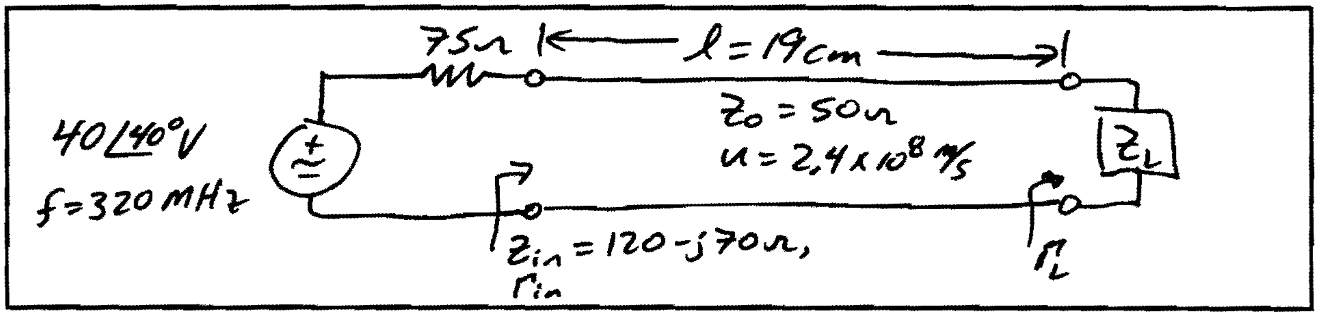
Monday, September 8, 2025 A

EE 381 Electric and Magnetic Fields Quiz #2 (Fall 2025)

Name __KEY A__

Instructions: Closed book & notes (see back). Place answers in indicated spaces and show all work for credit.

A lossless 19 cm-long transmission line (50 Ω characteristic impedance and 2.4×10^8 m/s velocity of propagation) is terminated with an unknown load. It is driven by a generator with voltage $40\angle 40°$ V and impedance 75 Ω operating at 320 MHz. If the input impedance is measured to be $120 - j70$ Ω, sketch a fully labeled drawing of this transmission line circuit. Then, find the phase constant, input & load reflection coefficients, load impedance, and standing wave ratio. **Express reflection coefficients in polar/phasor format w/ angles in degrees, and impedances in rectangular format.**



$$\beta = \frac{\omega}{u} = \frac{2\pi(320\times10^6)}{2.4\times10^8} = \underline{8.37758 \text{ rad/m}}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{(120-j70)-50}{(120-j70)+50} = \underline{\underline{0.53846 \angle -22.61986°}}$$

$$\Gamma_L = \Gamma_{in}\, e^{+j2\beta\ell} = (0.5385\angle -22.62°)\, e^{+j2(8.37758)(0.19)}$$

$$= \underline{0.53846 \angle 159.7801°}$$

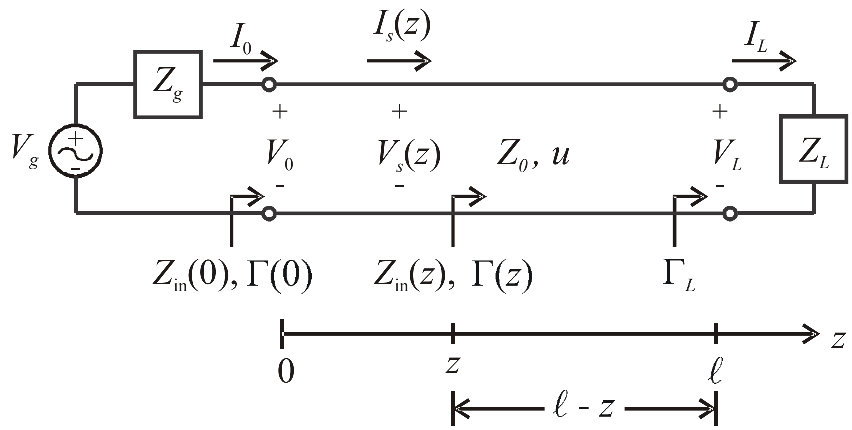
$$Z_L = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] = 50\left[\frac{1+0.5385\angle 159.78°}{1-0.5385\angle 159.78°}\right]$$

$$= \underline{15.4327 + j8.0898\,\Omega}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.53846}{1-0.53846} = \underline{3.333}$$

β = **8.3776 rad/m** Γ_{in} = **$0.5385\angle -22.62°$** Γ_L = **$0.5385\angle 159.78°$**

Z_L = **$15.433 + j\,8.0898$ Ω** SWR = **3.333**



$$u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ (m/s)}, \quad \beta = \frac{\omega}{u} = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \quad \text{(rad/m)}, \quad \lambda = \frac{u}{f} = \frac{2\pi}{\beta} \quad \text{(m)}$$

$$V_S(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z}[1+\Gamma(z)], \quad I_S(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z}[1-\Gamma(z)]$$

$$V_S(0) = V_0 = V_0^+ + V_0^-, \quad I_S(0) = I_0 = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1+\Gamma(0)} = 0.5(V_L + I_L Z_0)\, e^{+j\beta\ell} \text{ and } I_0^+ = \frac{V_0^+}{Z_0}$$

$$V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+ = 0.5(V_L - I_L Z_0)\, e^{-j\beta\ell} \text{ and } I_0^- = \frac{-V_0^-}{Z_0}.$$

$$\Gamma(z) = \frac{V_{\text{ref}}(z)}{V_{\text{inc}}(z)} = -\frac{I_{\text{ref}}(z)}{I_{\text{inc}}(z)} = \frac{Z_{\text{in}}(z) - Z_0}{Z_{\text{in}}(z) + Z_0} = \Gamma_L\, e^{-j2\beta(\ell - z)} \text{ for } 0 \le z \le \ell,$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{+j2\beta\ell} = \Gamma(0) e^{+j2\beta\ell} = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and } \Gamma_{\text{in}} = \Gamma(0) = \frac{Z_{\text{in}}(0) - Z_0}{Z_{\text{in}}(0) + Z_0} = \Gamma_L\, e^{-j2\beta\ell}$$

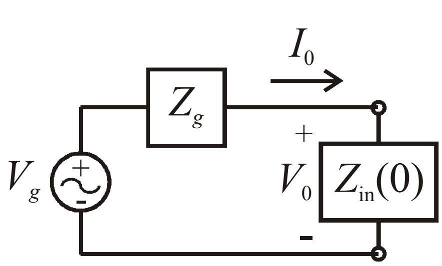
$$Z_{\text{in}}(z) = \frac{V(z)}{I(z)} = Z_0\left[\frac{Z_L + jZ_0 \tan[\beta(\ell - z)]}{Z_0 + jZ_L \tan[\beta(\ell - z)]}\right] = Z_0\left[\frac{1+\Gamma(z)}{1-\Gamma(z)}\right], \quad Z_L = Z_{\text{in}}(\ell) = \frac{V_L}{I_L} = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right]$$

$$Z_{\text{in}} = Z_{\text{in}}(0) = \frac{V(0)}{I(0)} = \frac{V_0}{I_0} = Z_0\left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right] = Z_0\left[\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}\right] = Z_0\left[\frac{1+\Gamma(0)}{1-\Gamma(0)}\right] = Z_0\left[\frac{1+\Gamma_{\text{in}}}{1-\Gamma_{\text{in}}}\right]$$

$$SWR = S = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$P_{\text{ave}}(z) = 0.5\,\text{Re}\{V(z) I^*(z)\} = 0.5\frac{|V_0^+|^2}{Z_0}\left[1-|\Gamma(z)|^2\right] = P_{\text{ave}}(0) = 0.5\,\text{Re}\{V_0 I^*{}_0\} = 0.5\frac{|V_0^+|^2}{Z_0}\left[1-|\Gamma(0)|^2\right]$$

$$= P_{\text{ave}}(\ell) = 0.5\,\text{Re}\{V_L I^*{}_L\}$$



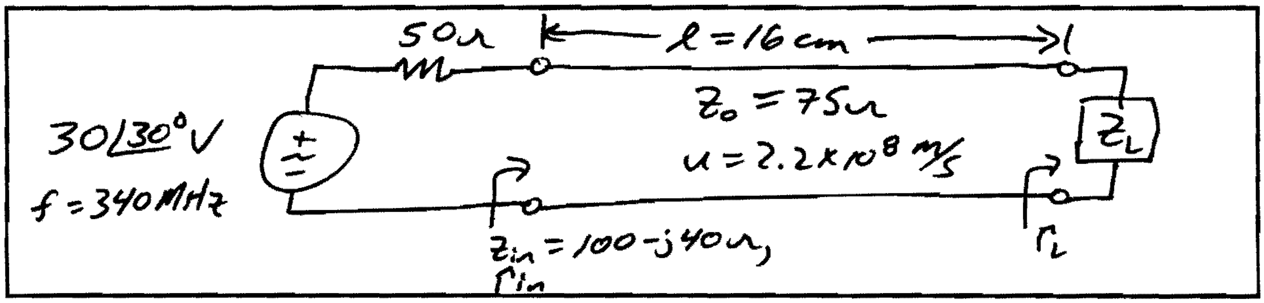
$$V_0 = V_g\left(\frac{Z_{\text{in}}(0)}{Z_g + Z_{\text{in}}(0)}\right) \text{ and } I_0 = \frac{V_g}{Z_g + Z_{\text{in}}(0)}.$$

EE 381 Electric and Magnetic Fields Quiz #2 (Fall 2025)

Name **KEY B**

Instructions: Closed book & notes (see back). Place answers in indicated spaces and show all work for credit.

A lossless 16 cm-long transmission line (75 Ω characteristic impedance and 2.2×10^8 m/s velocity of propagation) is terminated with an unknown load. It is driven by a generator with voltage $30\angle 30°$ V and impedance 50 Ω operating at 340 MHz. If the input impedance is measured to be 100 - j40 Ω, sketch a fully labeled drawing of this transmission line circuit. Then, find the phase constant, input & load reflection coefficients, load impedance, and standing wave ratio. **Express reflection coefficients in polar/phasor format w/ angles in degrees, and impedances in rectangular format.**



$$\beta = \omega / u = \frac{2\pi(340\times10^6)}{2.2\times10^8} = \underline{9.7104 \text{ rad/m}}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{(100 - j40) - 75}{(100 - j40) + 75} = \underline{0.26277\angle -45.1196°}$$

$$\Gamma_L = \Gamma_{in} e^{+j2\beta\ell} = (0.263\angle -45.1°)\, e^{j2(9.71)0.16}$$

$$= \underline{0.26277\angle 132.9167°}$$

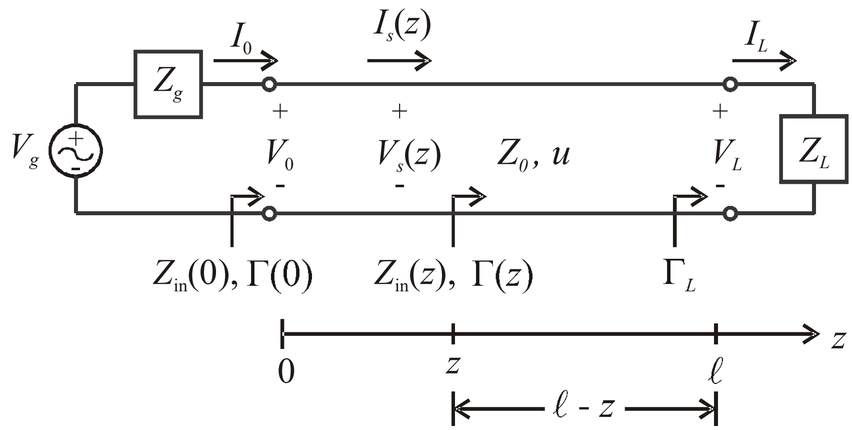
$$Z_L = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] = 75\left[\frac{1+0.263\angle 132.9°}{1-0.263\angle 132.9°}\right]$$

$$= \underline{48.9324 + j\,20.2293\,\Omega}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.26277}{1-0.26277} = \underline{1.7128}$$

β = **9.7104 rad/m** Γ_{in} = **0.2628∠-45.12°** Γ_L = **0.2628∠132.92°**

Z_L = **48.932 + *j* 20.229 Ω** SWR = **1.713**



$$u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ (m/s)}, \quad \beta = \frac{\omega}{u} = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \quad \text{(rad/m)}, \quad \lambda = \frac{u}{f} = \frac{2\pi}{\beta} \quad \text{(m)}$$

$$V_S(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z}[1+\Gamma(z)], \quad I_S(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z}[1-\Gamma(z)]$$

$$V_S(0) = V_0 = V_0^+ + V_0^-, \quad I_S(0) = I_0 = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1+\Gamma(0)} = 0.5(V_L + I_L Z_0)\, e^{+j\beta\ell} \text{ and } I_0^+ = \frac{V_0^+}{Z_0}$$

$$V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+ = 0.5(V_L - I_L Z_0)\, e^{-j\beta\ell} \text{ and } I_0^- = \frac{-V_0^-}{Z_0}.$$

$$\Gamma(z) = \frac{V_{\text{ref}}(z)}{V_{\text{inc}}(z)} = -\frac{I_{\text{ref}}(z)}{I_{\text{inc}}(z)} = \frac{Z_{\text{in}}(z) - Z_0}{Z_{\text{in}}(z) + Z_0} = \Gamma_L\, e^{-j2\beta(\ell - z)} \text{ for } 0 \le z \le \ell,$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{+j2\beta\ell} = \Gamma(0) e^{+j2\beta\ell} = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and } \Gamma_{\text{in}} = \Gamma(0) = \frac{Z_{\text{in}}(0) - Z_0}{Z_{\text{in}}(0) + Z_0} = \Gamma_L\, e^{-j2\beta\ell}$$

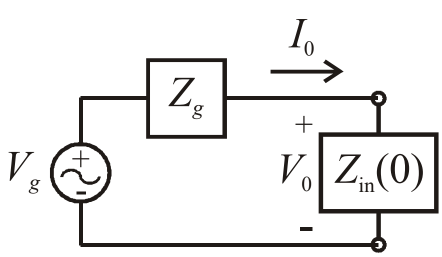
$$Z_{\text{in}}(z) = \frac{V(z)}{I(z)} = Z_0\left[\frac{Z_L + jZ_0 \tan[\beta(\ell - z)]}{Z_0 + jZ_L \tan[\beta(\ell - z)]}\right] = Z_0\left[\frac{1+\Gamma(z)}{1-\Gamma(z)}\right], \quad Z_L = Z_{\text{in}}(\ell) = \frac{V_L}{I_L} = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right]$$

$$Z_{\text{in}} = Z_{\text{in}}(0) = \frac{V(0)}{I(0)} = \frac{V_0}{I_0} = Z_0\left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right] = Z_0\left[\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}\right] = Z_0\left[\frac{1+\Gamma(0)}{1-\Gamma(0)}\right] = Z_0\left[\frac{1+\Gamma_{\text{in}}}{1-\Gamma_{\text{in}}}\right]$$

$$SWR = S = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$P_{\text{ave}}(z) = 0.5\,\text{Re}\left\{V(z) I^*(z)\right\} = 0.5\frac{|V_0^+|^2}{Z_0}\left[1 - |\Gamma(z)|^2\right] = P_{\text{ave}}(0) = 0.5\,\text{Re}\left\{V_0 I^*_0\right\} = 0.5\frac{|V_0^+|^2}{Z_0}\left[1 - |\Gamma(0)|^2\right]$$

$$= P_{\text{ave}}(\ell) = 0.5\,\text{Re}\left\{V_L I^*_L\right\}$$



$$V_0 = V_g\left(\frac{Z_{\text{in}}(0)}{Z_g + Z_{\text{in}}(0)}\right) \text{ and } I_0 = \frac{V_g}{Z_g + Z_{\text{in}}(0)}.$$