EE 381 Electric & Magnetic Fields Examination #2 (Fall 2025)

Name **KEY MM**

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheets in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) Given the electrostatic potential $V = 200\,x z^2$ (V) exists in a region of free space, determine:

a) the electric field $\bar{E}$,

Per (4.76),

$$\bar{E} = -\bar{\nabla} V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z}$$

$$= -\hat{a}_x \frac{\partial (200\,x z^2)}{\partial x} - \hat{a}_y \frac{\partial (200\,x z^2)}{\partial y} - \hat{a}_z \frac{\partial (200\,x z^2)}{\partial z}$$

$$= -200\,z^2 \hat{a}_x - 0 - (2)200\,x z \hat{a}_z$$

$\bar{E}$ = $-200\,z^2\,\hat{a}_x - 400\,x\,z\,\hat{a}_z$ **(V/m)**

b) electric flux density $\bar{D}$,

Per (4.35), $\bar{D} = \varepsilon_0 \bar{E} = 8.8541878\times10^{-12}(-200\,z^2 \hat{a}_x - 400\,x\,z\,\hat{a}_z)$

$\bar{D}$ = $-1.7708\,z^2\,\hat{a}_x - 3.54165\,x\,z\,\hat{a}_z$ **(nC/m²)**

c) volume charge density ρ_v, and

Per (4.43),

$$\rho_v = \bar{\nabla}\bullet\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial(-1.7708\times10^{-9} z^2)}{\partial x} + \frac{\partial (0)}{\partial y} + \frac{\partial(-3.54165\times10^{-9}\,x\,z)}{\partial z}$$

$$= -3.54165\times10^{-9}\,x\ (\text{C/m}^3)$$

volume charge density = $\rho_v = -3.54165\,x$ **(nC/m³)**

d) electrostatic energy density (extra credit)

Per (4.97),

$$w_E = 0.5\,\bar{D}\bullet\bar{E} = 0.5\varepsilon_0 |\bar{E}|^2 = 0.5(8.8541878\times10^{-12})[(-200\,z^2)^2 + (-400\,x\,z)^2]$$

$$= 0.5(8.8541878\times10^{-12})[40{,}000\,z^4 + 160{,}000\,x^2\,z^2]$$

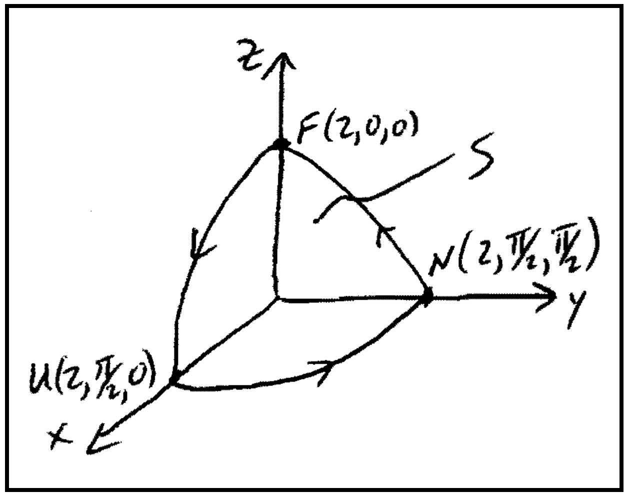
electrostatic energy density = **$w_E = 177.08\,z^4 + 708.33\,x^2\,z^2$ (nJ/m³)**

2) The surface S, defined by $r = 2$ m, $0 \le \theta \le \pi/2$, & $0 \le \phi \le \pi/2$, has corners at points F(2 m, 0, 0), U(2 m, π/2, 0), and N(2 m, π/2, π/2) in spherical coordinates. In box, sketch surface S labeling points F, U, & N. Determine the circulation of vector field $\overline{A} = \dfrac{4\sin\theta}{r^2}\hat{a}_r$ (alpacas/m^2) about the edge of S when going on path L from point $F \rightarrow U \rightarrow N \rightarrow F$. Also, find the outward flux of $\overline{A}$ through S.

Circulation = $\oint_L \overline{A} \bullet d\overline{\ell}$ where

$$\overline{A} \bullet d\overline{\ell} = \frac{4\sin\theta}{r^2}\hat{a}_r \bullet (dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi)$$

$$= \frac{4\sin\theta}{r^2}\,dr \quad \text{(alpacas/m)}$$



The path L is divided into three distinct sections:

$F \rightarrow U$ where $r = 2$ m ($\Rightarrow dr = 0$), θ changes, & $\phi = 0$;

$U \rightarrow N$ where $r = 2$ m ($\Rightarrow dr = 0$), $\theta = \pi/2$, & ϕ changes;

$N \rightarrow F$ where $r = 2$ m ($\Rightarrow dr = 0$), θ changes, & $\phi = \pi/2$;

Therefore, the circulation of about path L is

$$\oint_L \overline{A} \bullet d\overline{\ell} = \int_F^U \overline{A} \bullet d\overline{\ell} + \int_U^N \overline{A} \bullet d\overline{\ell} + \int_N^F \overline{A} \bullet d\overline{\ell}$$

$$= \int_{\theta=0}^{\pi/2} \frac{4\sin\theta}{2^2}\,dr + \int_{\phi=0}^{\pi/2} \frac{4\sin(\pi/2)}{2^2}\,dr + \int_{\theta=\pi/2}^{0} \frac{4\sin\theta}{2^2}\,dr = 0 + 0 + 0 = 0$$

The outward flux of $\overline{A}$ through S is $\text{flux}_A = \iint_S \overline{A} \bullet d\overline{s}$ where $d\overline{s} = d\overline{s}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$.

$$\text{flux}_A = \iint_S \frac{4\sin\theta}{r^2}\hat{a}_r \bullet r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r = \iint_S 4\sin^2\theta\,d\theta\,d\phi$$

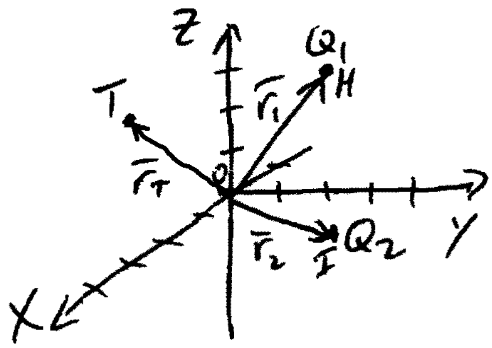
$$= 4\int_{\phi=0}^{\pi/2} d\phi \int_{\theta=0}^{\pi/2} \sin^2\theta\,d\theta = 4(\phi)\Big|_{\phi=0}^{\pi/2}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\phi=0}^{\pi/2}$$

$$= 4\left(\frac{\pi}{2} - 0\right)\left[\left(\frac{\pi/2}{2} - \frac{\sin\pi}{4}\right) - \left(\frac{0}{2} - \frac{\sin 0}{4}\right)\right] = 4\left(\frac{\pi}{2}\right)\left[\left(\frac{\pi}{4} - 0\right) - (0 - 0)\right]$$

$$= \frac{\pi^2}{2} = 4.9348 \quad \text{(alpacas)}$$

circulation$_A$ = **0** flux$_A$ = **4.9348 alpacas**

3) In free space, the point charges $Q_1 = 0.16$ μC and $Q_2 = -0.12$ μC are positioned at the Cartesian points $H(1, 2, 3)$ and $I(-1, 2, -1)$ respectively. Determine the electric field at the origin. Also, find the electrostatic potential of point $T(3, 0, 2)$ with respect to the origin. For extra credit, calculate how much electrostatic energy is stored in this charge assembly. All locations are in units of meters.



Define the various position vectors:

For Q_1, $\bar{r}_1 = \bar{r}_H = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$ (m),

for Q_2, $\bar{r}_2 = \bar{r}_I = -\hat{a}_x + 2\hat{a}_y - \hat{a}_z$ (m),

for point T, $\bar{r}_T = 3\hat{a}_x + 2\hat{a}_z$ (m), and

for origin, $\bar{r}_O = 0$.

To find the electric field at the origin due to Q_1 & Q_2, use (4.12)

$$\bar{E}_O = \frac{Q_1(\bar{r}_O - \bar{r}_1)}{4\pi\varepsilon_0 |\bar{r}_O - \bar{r}_1|^3} + \frac{Q_2(\bar{r}_O - \bar{r}_2)}{4\pi\varepsilon_0 |\bar{r}_O - \bar{r}_2|^3}$$

$$= \frac{0.16\times10^{-6}[0-(\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)]}{4\pi\varepsilon_0 |0-(\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)|^3} + \frac{-0.12\times10^{-6}[0-(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)]}{4\pi\varepsilon_0 |0-(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)|^3}$$

$$= \frac{0.16\times10^{-6}(-\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z)}{4\pi\varepsilon_0 [(-1)^2 + 2^2 + 3^2]^{3/2}} + \frac{0.12\times10^{-6}(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)}{4\pi\varepsilon_0 [1^2 + (-2)^2 + 1^2]^{3/2}}$$

$$= \frac{0.16\times10^{-6}(-\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z)}{4\pi\, 8.8541878\times10^{-12}(14)^{3/2}} + \frac{0.12\times10^{-6}(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)}{4\pi\, 8.8541878\times10^{-12}(6)^{3/2}}$$

$$= 27.45192(-\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z) + 73.383615(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)$$

$$= (-27.45192 - 73.38362)\hat{a}_x + (-54.90384 + 146.76723)\hat{a}_y + (-82.35576 - 73.38362)\hat{a}_z$$

$$= -100.8355\hat{a}_x + 91.8634\hat{a}_y - 155.7394\hat{a}_z \quad \text{(V/m)}$$

$$\bar{E}_O = \underline{-100.8355\hat{a}_x + 91.8634\hat{a}_y - 155.7394\hat{a}_z \quad \text{(V/m)}}$$

3) cont.

To find the electrostatic potential of point T with respect to the origin, i.e., $V_{TO} = V_T - V_O$, use (4.66) to find the absolute potential at the origin (V_O) and at point T (V_T) due to Q_1 & Q_2.

$$V_O = \frac{Q_1}{4\pi\varepsilon_0 |\bar{r}_O - \bar{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\bar{r}_O - \bar{r}_2|}$$

$$= \frac{0.16\times10^{-6}}{4\pi\, 8.8541878\times10^{-12}(14)^{3/2}} + \frac{-0.12\times10^{-6}}{4\pi\, 8.8541878\times10^{-12}(6)^{3/2}}$$

$$= 384.3268 - 440.30169 = -55.97489 \;\; \text{(V)}$$

$$V_T = \frac{Q_1}{4\pi\varepsilon_0 |\bar{r}_T - \bar{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\bar{r}_T - \bar{r}_2|}$$

$$= \frac{0.16\times10^{-6}}{4\pi\varepsilon_0 |(3\hat{a}_x + 2\hat{a}_z) - (\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)|} + \frac{-0.12\times10^{-6}}{4\pi\varepsilon_0 |(3\hat{a}_x + 2\hat{a}_z) - (-\hat{a}_x + 2\hat{a}_y - \hat{a}_z)|}$$

$$= \frac{0.16\times10^{-6}}{4\pi\varepsilon_0 [2^2 + (-2)^2 + (-1)^2]^{1/2}} + \frac{-0.12\times10^{-6}}{4\pi\varepsilon_0 [4^2 + (-2)^2 + 3^2]^{1/2}}$$

$$= \frac{0.16\times10^{-6}}{4\pi\, 8.8541878\times10^{-12}(3)} + \frac{-0.12\times10^{-6}}{4\pi\, 8.8541878\times10^{-12}(29)^{1/2}}$$

$$= 479.339769 - 200.27511 = 279.06466 \;\; \text{(V)}$$

$V_{TO} = V_T - V_O = 279.06466 - (-55.97489)$ $\Rightarrow$ **V_{TO} = 335.03955 V.**

Extra credit- Adapting (4.84), $W_E = W_1 + W_2 = 0 + Q_2 V_{21} = Q_2 V_{21}$. Use (4.65) to find V_{21}

$$V_{21} = \frac{Q_1}{4\pi\varepsilon_0 |\bar{r}_2 - \bar{r}_1|} = \frac{0.16\times10^{-6}}{4\pi\varepsilon_0 |(-\hat{a}_x + 2\hat{a}_y - \hat{a}_z) - (\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)|}$$

$$= \frac{0.16\times10^{-6}}{4\pi\varepsilon_0 [(-2)^2 + 0^2 + (-4)^2]^{1/2}} = \frac{0.16\times10^{-6}}{4\pi\, 8.8541878\times10^{-12}(20)^{1/2}}$$

$$= 321.5484286 \;\; \text{(V)}$$

$W_E = -0.12\times10^{-6}(321.5484286)$ $\Rightarrow$ **$W_E = -3.85858\times10^{-5}$ J = -38.5858 μJ.**

V_{TO} = **335.04 V** (extra credit) electrostatic energy = **W_E = -38.586 μJ**