EE 381 Electric & Magnetic Fields Examination #1 (Fall 2025)

Name **KEY A**

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheets in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) A coaxial transmission line (TL) is made with conductors (ε_0, μ_0, and $\sigma_c = 8\times10^6$ S/m) with the inner conductor having a diameter of 3 mm and the shield having an inner diameter of 10 mm and thickness of 0.5 mm separated by plastic insulation ($2.1\varepsilon_0$, μ_0, and $\sigma = 6\times10^{-6}$ S/m). Determine the skin depth and per-unit-length parameters R, L, C, and G at 9 GHz. Then, determine the characteristic impedance and propagation constant of the TL. Put complex quantities in rectangular format.

Per Table 11.1,

$$\delta_s = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi(9\times10^9)4\pi\times10^{-7}(8\times10^6)}} \Rightarrow \underline{\delta_s = 1.87566\times10^{-6}\text{ m} = 1.876\ \mu\text{m}}.$$

Note: The lead shield thickness 0.5 mm >> δ_s. Also, $2a$ = 3 mm and $2b$ = 10 mm.

Using equations from Table 11.1,

$$R = \frac{1}{2\pi\delta_s\sigma_c}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{2\pi(1.87566\times10^{-6})(8\times10^6)}\left(\frac{1}{0.0015}+\frac{1}{0.005}\right)$$

$$\Rightarrow \underline{R = 9.19239\ \Omega/\text{m}}$$

$$L = \frac{\mu}{2\pi}\ln(b/a) = \frac{4\pi\times10^{-7}}{2\pi}\ln(10/3) \Rightarrow \underline{L = 2.40795\times10^{-7}\text{ H/m} = 240.795\text{ nH/m}}$$

$$G = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi(6\times10^{-6})}{\ln(10/3)} \Rightarrow \underline{G = 3.131226\times10^{-5}\text{ S/m} = 31.312\ \mu\text{S/m}}$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi(2.1)8.8541878\times10^{-12}}{\ln(10/3)} \Rightarrow \underline{C = 9.70356\times10^{-11}\text{ F/m} = 97.036\text{ pF/m}}$$

Per (11.11),

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(9.1924 + j2\pi(9{\bullet}10^9)2.4{\bullet}10^{-7})(3.1{\bullet}10^{-5} + j2\pi(9{\bullet}10^9)9.7{\bullet}10^{-11})}$$

$$\Rightarrow \underline{\gamma = 0.09305 + j273.3451\ 1/\text{m}}.$$

Per (11.19), $$Z_C = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{9.1924 + j2\pi(9{\bullet}10^9)2.4{\bullet}10^{-7}}{3.1{\bullet}10^{-5} + j2\pi(9{\bullet}10^9)9.7{\bullet}10^{-11}}} \Rightarrow \underline{Z_0 = 49.81473 - j0.01667\ \Omega}.$$

δ_s = **1.876 μm** R = **9.1924 Ω/m** L = **240.795 nH/m** C = **97.036 pF/m**

G = **31.312 μS/m** Z_C = **49.81473 - j0.01667 Ω** γ = **0.09305 + j273.3451 1/m**

2) Given vectors $\bar{H} = -3\,\hat{a}_x + 5\,\hat{a}_y - 7\,\hat{a}_z$ and $\bar{I} = 6\,\hat{a}_r$ and the spherical point $P(5, 0.36\pi, 1.7\pi)$, find:

a) the point P in Cartesian coordinates,

$$\text{Per (2.22),}\quad x = r\sin(\theta)\cos(\phi) = 5\sin(0.36\pi)\cos(1.7\pi) = 2.6592$$

$$y = r\sin(\theta)\sin(\phi) = 5\sin(0.36\pi)\sin(1.7\pi) = -3.6601$$

$$z = r\cos(\theta) = 5\cos(0.36\pi) = 2.1289$$

$P_{\text{Cartesian}} = \underline{(2.6592, -3.6601, 2.1289)}$

b) vector $\bar{I}$ in Cartesian coordinates at point P,

Per (2.24), use $\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$ in

$$\bar{I} = 6\hat{a}_r = 6\left(\sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z\right)|_P$$

$$= 6\left(\sin 0.36\pi\cos 1.7\pi\,\hat{a}_x + \sin 0.36\pi\sin 1.7\pi\,\hat{a}_y + \cos 0.36\pi\,\hat{a}_z\right)$$

$$= 3.191\hat{a}_x - 4.3921\hat{a}_y + 2.5547\hat{a}_z$$

$\bar{I}_{\text{Cart}} = \underline{3.191\hat{a}_x - 4.3921\hat{a}_y + 2.5547\hat{a}_z}$

c) the smallest angle (degrees) between $\bar{H}$ and $\bar{I}$ at point P, and

Use (1.15), $\bar{H}\bullet\bar{I} = |H||\bar{I}|\cos\theta_{HI}$ where

$$\bar{H}\bullet\bar{I} = (-3\hat{a}_x + 5\hat{a}_y - 7\hat{a}_z)\bullet(3.191\hat{a}_x - 4.3921\hat{a}_y + 2.5547\hat{a}_z)$$

$$= -3(3.191) + 5(-4.3921) - 7(2.5547) = -49.4165,$$

$$|H| = \sqrt{(-3)^2 + 5^2 + (-7)^2} = \sqrt{83} = 9.1104, \text{ and}$$

$$|\bar{I}| = \sqrt{(3.191)^2 + (-4.3921)^2 + (2.5547)^2} = 6.$$

$$\text{Next, } \cos\theta_{HI} = \frac{\bar{H}\bullet\bar{I}}{|H||\bar{I}|} \Rightarrow \theta_{HI} = \cos^{-1}\left(\frac{\bar{H}\bullet\bar{I}}{|H||\bar{I}|}\right) = \cos^{-1}\left(\frac{-49.4165}{9.1104(6)}\right)$$

$\theta_P = \underline{\mathbf{154.693°}}$

d) the **vector** component of $\bar{I}$ in the direction of $\bar{H}$ at point P (express in Cartesian coordinates)

Use (1.35), $\bar{I}_H = (\bar{I}\bullet\hat{a}_H)\hat{a}_H$ where

$$\hat{a}_H = \frac{\bar{H}}{|\bar{H}|} = \frac{-3\hat{a}_x + 5\hat{a}_y - 7\hat{a}_z}{9.1104} = -0.329293\hat{a}_x + 0.548821\hat{a}_y - 0.76835\hat{a}_z$$

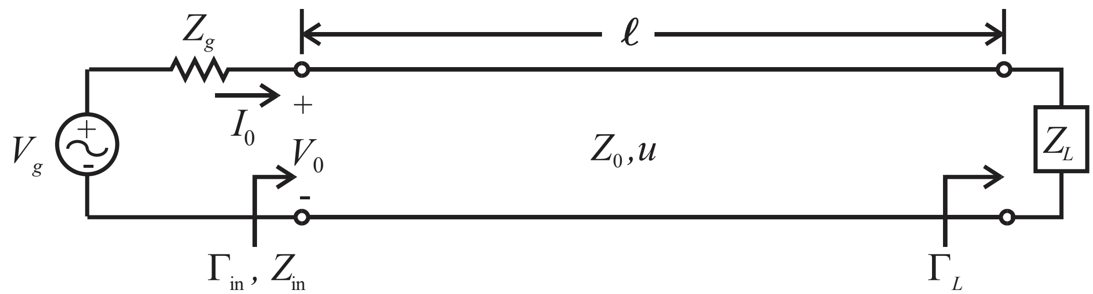
$$\bar{I}\bullet\hat{a}_H = (3.191\hat{a}_x - 4.3921\hat{a}_y + 2.5547\hat{a}_z)\bullet(-0.329293\hat{a}_x + 0.548821\hat{a}_y - 0.76835\hat{a}_z)$$

$$= 3.191(-0.329293) - 4.3921(0.548821) + 2.5547(-0.76835) = -5.4241$$

$$\bar{I}_H = (-5.4241)(-0.329293\hat{a}_x + 0.548821\hat{a}_y - 0.76835\hat{a}_z)$$

$\bar{I}_H = \underline{1.786\hat{a}_x - 2.977\hat{a}_y + 4.168\hat{a}_z}$

3) For the lossless transmission (TL) circuit shown, $Z_0 = 75\ \Omega$, $u = 2.25 \times 10^8$ m/s, $\ell = 27.9$ cm, $f = 2.1$ GHz, $Z_L = 30 + j\,20\ \Omega$, $Z_g = 50\ \Omega$, and $V_g = 100\angle 0^\circ$ (V). Calculate the wavelength and electrical length (in radians) of the TL. Next, find the load reflection coefficient Γ_L, input impedance, input phasor current & voltage, and time-average power delivered to the load. **Express phasor current & voltage and Γ as $A\angle\theta$ (w/ angle in degrees) and impedance in rectangular format (e.g., $a + j\,b$).**



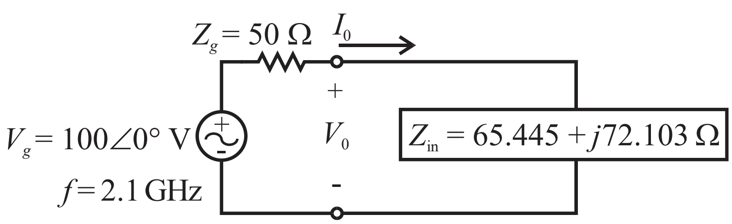
$\lambda = u/f = 2.25 \times 10^8 / 2.1 \times 10^9$ $\Rightarrow$ $\lambda = 0.10714\text{ m} = 10.714\text{ cm}$

$\beta\ell = (2\pi/\lambda)\,\ell = (2\pi/0.10714)\,0.279$ $\Rightarrow$ $\beta\ell = 16.3614$ rad/m

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 + j20) - 75}{(30 + j20) + 75}$$ $\Rightarrow$ $\Gamma_L = 0.4607\angle 145.253^\circ$

$$\Gamma_{in} = \Gamma_L\, e^{-j2\beta\ell} = (0.4607\angle 145.253^\circ)\, e^{-j2(16.3614)}$$ $\Rightarrow$ $\Gamma_{in} = 0.4607\angle 70.373^\circ$

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 75\left[\frac{1 + (0.4607\angle 70.373^\circ)}{1 - (0.4607\angle 70.373^\circ)}\right]$$ $\Rightarrow$ $Z_{in} = 65.445 + j\,72.103\ \Omega$



$$I_0 = \frac{V_g}{Z_g + Z_{in}} = \frac{100\angle 0^\circ}{50 + (65.445 + j72.103)}$$ $\Rightarrow$ $I_0 = 0.73469\angle\text{-}31.987^\circ$ A

$$V_0 = V_g \frac{Z_{in}}{Z_g + Z_{in}} = 100\angle 0^\circ \frac{65.445 + j72.103}{50 + (65.445 + j72.103)}$$ $\Rightarrow$ $V_0 = 71.5405\angle 15.784^\circ$ V

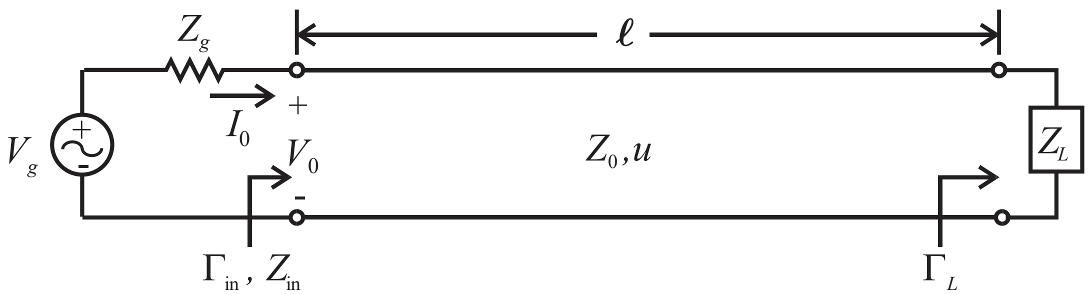
For a lossless TL, the input power and load power are the same.

$P_L = P_{in} = 0.5\,\mathrm{Re}\{V_0\, I_0^*\} = 0.5\,\mathrm{Re}\{(71.54\angle 15.78^\circ)(0.7347\angle 31.99^\circ)\}$ $\Rightarrow$ $P_L = 17.663$ W.

λ = 10.71 cm $\beta\ell$ = 16.361 rad/m Γ_L = $0.4607\angle 145.253^\circ$ Z_{in} = $65.445 + j\,72.103\ \Omega$

I_0 = $0.7347\angle\text{-}31.99^\circ$ A V_0 = $71.54\angle 15.78^\circ$ V P_L = 17.663 W

4) For the lossless transmission circuit shown, $Z_0 = 50\ \Omega$, $u = 2.4 \times 10^8$ m/s, $\ell = 2.7\lambda$, $f = 980$ MHz, $Z_{in} = 130 - j\,120\ \Omega$, $Z_g = 75\ \Omega$, and $V_g = 40\ \angle 30°$ (V). Using a **Smith chart (show & label all work)**, find the input and load reflection coefficients, input admittance, load impedance, and standing wave ratio. **Express reflection coefficients in polar form w/ angle in degrees and admittances & impedances in rectangular format**.



- Find normalized input impedance $z_{in} = Z_{in}/Z_0 = (130 - j\,120)/50$ **= 2.6 - *j*2.4 Ω/Ω**. Plot z_{in}.
- Use compass to draw circle centered on Smith chart through z_{in} and make marks on 'REFL. COEFF, Vor I' and 'SWR (VSWR)' scales. Read **$|\Gamma_{in}| = 0.67$** and **SWR = 5**.
- Use straight edge to draw radial line from center of Smith chart to outer rings. On the 'ANGLE OF REFLECTION COEFFICIENT' scale, read **$\angle\Gamma_{in} = -22.6°$**. Put together to get $\Rightarrow$ **$\Gamma_{in} = 0.67\angle{-22.6°}$.**
- Calculate TL length $\ell/\lambda = 2.7\lambda\ /\ \lambda = 2.7 \Rightarrow 0.2$. Move 0.2λ on an arc/circle of constant radius $|\Gamma|$ in the 'WAVELENGTHS TOWARD LOAD' direction (CCW) from z_{in} to z_L &/or Γ_L point. Read **$z_L = 0.26 + j0.53\ \Omega/\Omega$** and **$\angle\Gamma_L = 121.4°$** $\Rightarrow$ **$\Gamma_L = 0.67\angle 121.4°$.**
- Calculate load impedance as $Z_L = z_L Z_0 = (0.26 + j0.53)\,50 \Rightarrow$ **$Z_L = 13 + j26.5\ \Omega$**.
- Move 180° on the circle of constant radius $|\Gamma|$ from z_{in} to y_{in}. Read the normalized input admittance **$y_{in} = 0.208 + j0.193$ S/S**.
- Calculate $Y_{in} = y_{in}\ Y_0 = (0.208 + j0.193)/50 = \Rightarrow$ **$Y_{in} = 0.00416 + j\,0.00386$ S.**

Γ_{in} = **$0.67\angle{-22.6°}$** Γ_L = **$0.67\angle 121.4°$** Y_{in} = **$4.16 + j\,3.86$ mS**

Z_L = **$13 + j\,26.5\ \Omega$** SWR = **5**

Simple Smith Chart