

EE 381 Electric and Magnetic Fields Quiz #9 (Fall 2024)Name Key AInstructions: Open book & notes. Use units/notation as given in class. Put answers in indicated spaces and **show all** work for credit.

In a volume of oil-contaminated sea water ($\epsilon = 32\epsilon_0$, $\sigma = 4 \text{ S/m}$), the potential is $V = 360x^2 - 90x + 80 \text{ V}$. Find the electric field, electric flux density, current density, and polarization vectors as well as the free volume charge density when $x = 0.4 \text{ m}$.

$$(4.76) \quad \vec{E} = -\vec{\nabla}V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z}$$

$$= -\hat{a}_x (720x - 90) \text{ V/m}$$

$$\vec{E}(x=0.4 \text{ m}) = -\hat{a}_x [720(0.4) - 90] = -\hat{a}_x 198 \text{ V/m}$$

$$(5.34) \quad \vec{D} = \epsilon \vec{E} = 32(8.8541878 \times 10^{-12})(-\hat{a}_x 198)$$

$$\vec{D}(x=0.4 \text{ m}) = -\hat{a}_x 5.610 \times 10^{-8} \text{ C/m}^2$$

$$(5.11) \quad \vec{J} = \sigma \vec{E} = 4(-\hat{a}_x 198) \Rightarrow \vec{J}(0.4 \text{ m}) = -\hat{a}_x 792 \text{ A/m}^2$$

$$(5.31) \quad \vec{P} = \vec{D} - \epsilon_0 \vec{E} = -\hat{a}_x 5.61 \times 10^{-8} - 8.8541878 \times 10^{-12}(-\hat{a}_x 198)$$

$$\vec{P}(0.4 \text{ m}) = -\hat{a}_x 5.4347 \times 10^{-8} \text{ C/m}^2$$

$$(4.43) \quad \rho_v = \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot [-\hat{a}_x 32\epsilon_0(720x - 90)]$$

$$(5.34) \quad = -\frac{\partial}{\partial x} [32\epsilon_0(720x - 90)] = -23040\epsilon_0$$

$$= -23040(8.8541878 \times 10^{-12}) = -2.0400 \times 10^{-7} \text{ C/m}^3$$

(constant)

electric field vector = $\vec{E} = -\hat{a}_x 198 \text{ (V/m)}$

electric flux density vector = $\vec{D} = -\hat{a}_x 56.10 \text{ (nC/m}^2\text{)}$

current density vector = $\vec{J} = -\hat{a}_x 792 \text{ (A/m}^2\text{)}$

polarization vector = $\vec{P} = -\hat{a}_x 54.347 \text{ (nC/m}^2\text{)}$

free volume charge density = $\rho_v = -204.00 \text{ (nC/m}^3\text{)}$

EE 381 Electric and Magnetic Fields Quiz #9 (Fall 2024)

Name Key BInstructions: Open book & notes. Use units/notation as given in class. Put answers in indicated spaces and show all work for credit.

In a volume of oil-contaminated sea water ($\epsilon = 26\epsilon_0$, $\sigma = 2 \text{ S/m}$), the potential is $V = 350y^2 - 70y + 60 \text{ V}$. Find the electric field, electric flux density, current density, and polarization vectors as well as the free volume charge density when $y = 0.3 \text{ m}$.

$$(4.76) \quad \vec{E} = -\vec{\nabla}V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z} \\ = -\hat{a}_y \frac{\partial}{\partial y} (350y^2 - 70y + 60) = -\hat{a}_y (700y - 70) \text{ V/m}$$

$$\vec{E}(y=0.3\text{m}) = -\hat{a}_y [700(0.3) - 70] = -\hat{a}_y 140 \text{ V/m}$$

$$(5.34) \quad \vec{D} = \epsilon \vec{E} \Rightarrow \vec{D}(0.3\text{m}) = 26 (8.8541878 \times 10^{-12}) (-\hat{a}_y 140) \\ = -\hat{a}_y 3.2229 \times 10^{-8} \text{ C/m}^2$$

$$(5.11) \quad \vec{J} = \sigma \vec{E} \Rightarrow \vec{J}(0.3\text{m}) = 2 (-\hat{a}_y 140) = -\hat{a}_y 280 \text{ A/m}^2$$

$$(5.31) \quad \vec{P} = \vec{D} - \epsilon_0 \vec{E} \Rightarrow \vec{P}(0.3\text{m}) = -\hat{a}_y 3.2229 \times 10^{-8} + \hat{a}_y 8.8542 \times 10^{-12} (140) \\ = -\hat{a}_y 3.098966 \times 10^{-8} \text{ C/m}^2$$

$$(4.43) + (5.34) \quad \rho_v = \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot [-\hat{a}_y 26\epsilon_0 (700y - 70)] \\ = -\frac{\partial}{\partial y} (26\epsilon_0 (700y - 70)) = -18200\epsilon_0 \\ = -18200 (8.8541878 \times 10^{-12}) = -1.61146 \times 10^{-7} \text{ C/m}^3 \\ \text{(constant)}$$

electric field vector = $\vec{E} = -\hat{a}_y 140 \text{ (V/m)}$

electric flux density vector = $\vec{D} = -\hat{a}_y 32.229 \text{ (nC/m}^2\text{)}$

current density vector = $\vec{J} = -\hat{a}_y 280 \text{ (A/m}^2\text{)}$

polarization vector = $\vec{P} = -\hat{a}_y 30.990 \text{ (nC/m}^2\text{)}$

free volume charge density = $\rho_v = -161.146 \text{ (nC/m}^3\text{)}$