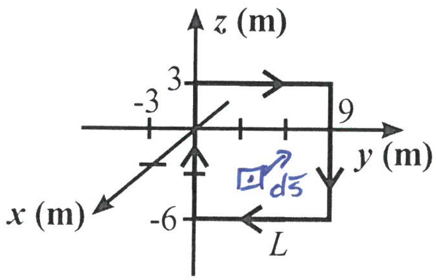
Monday, October 21, 2024 A

EE 381 Electric and Magnetic Fields Quiz #7 (Fall 2024)

Name Key A

Instructions: Open book & notes. Place answers in indicated spaces and **show all** work for credit.

Calculate the curl and divergence of the vector field $\bar{N} = 7\rho\sin\phi\,\hat{a}_\rho - 3\rho\,\hat{a}_\phi + 6z\,\hat{a}_z$ (Naps). Then, determine the circulation of $\bar{N}$ about the contour L shown.



By RHR, $d\bar{s} = -d\bar{s}_x = d\bar{s}_\phi$

$$\text{div}\,\bar{N} = \bar{\nabla}\cdot\bar{N} = \frac{1}{\rho}\frac{\partial(\rho N_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial N_\phi}{\partial\phi} + \frac{\partial N_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial(7\rho^2\sin\phi)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(-3\rho)}{\partial\phi} + \frac{\partial(6z)}{\partial z}$$

$$= \frac{14\rho\sin\phi}{\rho} + \frac{0}{\rho} + 6$$

$$\underline{\text{div}\,\bar{N} = 14\sin\phi + 6 \text{ (Naps/m)}}$$

$$\text{curl}\,\bar{N} = \bar{\nabla}\times\bar{N} = \hat{a}_\rho\left[\frac{1}{\rho}\frac{\partial N_z}{\partial\phi} - \frac{\partial N_\phi}{\partial z}\right] + \hat{a}_\phi\left[\frac{\partial N_\rho}{\partial z} - \frac{\partial N_z}{\partial\rho}\right] + \hat{a}_z\left[\frac{1}{\rho}\frac{\partial(\rho N_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial N_\rho}{\partial\phi}\right]$$

$$= \hat{a}_\rho\left[\frac{1}{\rho}\underbrace{\frac{\partial(6z)}{\partial\phi}}_{\to 0} - \underbrace{\frac{\partial(-3\rho)}{\partial z}}_{\to 0}\right] + \hat{a}_\phi\left[\underbrace{\frac{\partial(7\rho\sin\phi)}{\partial z}}_{\to 0} - \underbrace{\frac{\partial(6z)}{\partial\rho}}_{\to 0}\right] + \hat{a}_z\frac{1}{\rho}\left[\frac{\partial(-3\rho^2)}{\partial\rho} - \frac{\partial(7\rho\sin\phi)}{\partial\phi}\right]$$

$$\text{curl}\,\bar{N} = \hat{a}_z\frac{1}{\rho}\left[-6\rho - 7\rho\cos\phi\right] = \underline{-\hat{a}_z(6 + 7\cos\phi)\left(\frac{\text{Naps}}{\text{m}}\right)}$$

$$\text{Circulation} = \oint_L \bar{N}\cdot d\bar{\ell} = \iint_S (\bar{\nabla}\times\bar{N})\cdot d\bar{s} \text{ by Stoke's Thm}$$

$\Rightarrow \bar{N}$ is well behaved on S, $d\bar{s} = -d\bar{s}_x = -\hat{a}_x\,dy\,dz$

$$\Rightarrow \text{Circulation} = \iint_S -\hat{a}_z(6+7\cos\phi)\cdot -\hat{a}_x\,dy\,dz$$

$$= \underline{\underline{0}} \quad \text{since } \hat{a}_z\cdot\hat{a}_x = 0$$

(or $\hat{a}_z\cdot\hat{a}_\phi = 0$)

div $\bar{N}$ = $14\sin\phi + 6 \left(\frac{\text{Naps}}{\text{m}}\right)$ curl $\bar{N}$ = $-\hat{a}_z(6 + 7\cos\phi)\left(\frac{\text{Naps}}{\text{m}}\right)$

Circulation $\bar{N}$ about L = 0

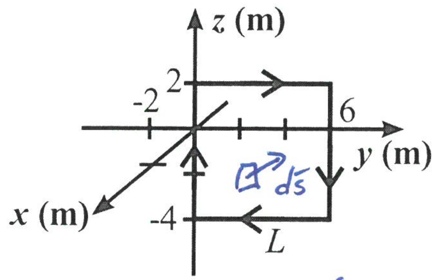
Monday, October 21, 2024 B

EE 381 Electric and Magnetic Fields Quiz #7 (Fall 2024)

Name Key B

Instructions: Open book & notes. Place answers in indicated spaces and **show all** work for credit.

Calculate the curl and divergence of the vector field $\bar{S} = 2\rho\sin\phi\,\hat{a}_\rho + 6\rho\,\hat{a}_\phi - 7z\,\hat{a}_z$ (Siestas). Then, determine the circulation of $\bar{S}$ about the contour L shown.



$$\operatorname{div}\bar{S} = \bar{\nabla}\cdot\bar{S} = \frac{1}{\rho}\frac{\partial(\rho S_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial S_\phi}{\partial\phi} + \frac{\partial S_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial(2\rho^2\sin\phi)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(6\rho)}{\partial\phi} + \frac{\partial(-7z)}{\partial z}$$

$$= \frac{4\rho\sin\phi}{\rho} + 0 - 7$$

By RHR, $d\bar{s} = -d\bar{s}_x = d\bar{s}_\phi$

$$\underline{\operatorname{div}\bar{S} = 4\sin\phi - 7 \text{ (Siestas/m)}}$$

$$\operatorname{curl}\bar{S} = \bar{\nabla}\times\bar{S} = \hat{a}_\rho\left[\frac{1}{\rho}\frac{\partial S_z}{\partial\phi} - \frac{\partial S_\phi}{\partial z}\right] + \hat{a}_\phi\left[\frac{\partial S_\rho}{\partial z} - \frac{\partial S_z}{\partial\rho}\right] + \hat{a}_z\frac{1}{\rho}\left[\frac{\partial(\rho S_\phi)}{\partial\rho} - \frac{\partial S_\rho}{\partial\phi}\right]$$

$$= \hat{a}_\rho\left[\frac{1}{\rho}\underbrace{\frac{\partial(-7z)}{\partial\phi}}_{0} - \underbrace{\frac{\partial(6\rho)}{\partial z}}_{0}\right] + \hat{a}_\phi\left[\underbrace{\frac{\partial(2\rho\sin\phi)}{\partial z}}_{0} - \underbrace{\frac{\partial(-7z)}{\partial\rho}}_{0}\right] + \hat{a}_z\frac{1}{\rho}\left[\frac{\partial(6\rho^2)}{\partial\rho} - \frac{\partial(2\rho\sin\phi)}{\partial\phi}\right]$$

$$= \hat{a}_z\frac{1}{\rho}\left[12\rho - 2\rho\cos\phi\right] = \underline{\hat{a}_z(12 - 2\cos\phi)\left(\frac{\text{Siestas}}{\text{m}}\right)}$$

$$\text{Circulation} = \oint_L \bar{S}\cdot d\bar{\ell} = \iint_S (\bar{\nabla}\times\bar{S})\cdot d\bar{s} \text{ by Stoke's Thm}$$

$\Rightarrow \bar{S}$ is well behaved on S, $d\bar{s} = -d\bar{s}_x = -\hat{a}_x\,dy\,dz$

$$\text{Circulation} = \iint_S \hat{a}_z(12 - 2\cos\phi)\cdot -\hat{a}_x\,dy\,dz$$

$$= \underline{\underline{0}} \text{ since } \hat{a}_z\cdot\hat{a}_x = 0$$

$$(\text{or } \hat{a}_z\cdot\hat{a}_\phi = 0)$$

div $\bar{S}$ = $4\sin\phi - 7\left(\frac{\text{Siestas}}{\text{m}}\right)$ curl $\bar{S}$ = $\hat{a}_z(12 - 2\cos\phi)\left(\frac{\text{Siestas}}{\text{m}}\right)$

Circulation $\bar{S}$ about L = 0