EE 381 Electric and Magnetic Fields Quiz #6 (Fall 2024)

Name Key A

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

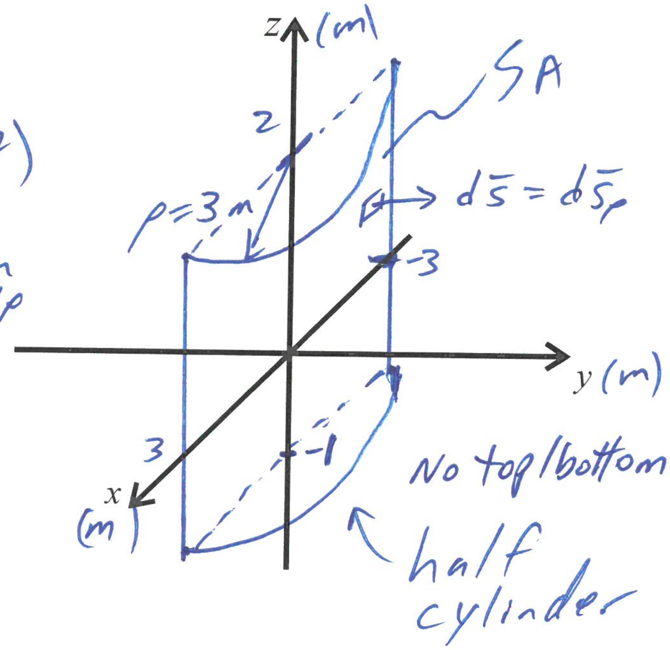
On the provided axes, draw a fully labeled sketch of the surface S_A defined by $\rho = 3$ m, $0 \le \phi < \pi$, $-1 \le z \le 2$ m. Next, find the outward (from origin) flux of the vector $\bar{L} = 2\rho^2 z^2\,\hat{a}_\rho + 4\sqrt{\rho}\,z^3\coth(\phi)\,\hat{a}_\phi - 8\cos(\phi)z\,\hat{a}_z$ (lemons/m^2) through S_A.

S_A is $\underline{\text{NOT}}$ a closed surface.

$d\bar{s} = d\bar{s}_\rho = \rho\, d\phi\, dz\, \hat{a}_\rho \quad (m^2)$

$\bar{L} \cdot d\bar{s}_\rho = (2\rho^2 z^2 \hat{a}_\rho + \ldots) \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$

$= 2\rho^3 z^2\, d\phi\, dz$



$$\psi_{SA} = \iint_{\substack{S_A \\ (\rho = 3m)}} \bar{L} \cdot d\bar{s}_\rho$$

$$= \int_{z=-1}^{2} \int_{\phi=0}^{\pi} 2(3)^3 z^2\, d\phi\, dz = 54 \int_{z=-1}^{2} z^2\, dz \int_{\phi=0}^{\pi} d\phi$$

$$= 54\ \frac{z^3}{3}\Big|_{-1}^{3}\ \phi\Big|_{0}^{\pi} = 54\left(\frac{8}{3} + \frac{1}{3}\right)(\pi - 0) = 162\pi$$

$\underline{\psi_{SA} = 508.938 \text{ lemons}}$

$(\text{Flux})_{SA} =$ 508.938 (lemons)

Rectangular/Cartesian Coordinates (x, y, z)

$$d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z \text{ (m)} \qquad \bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z \text{ (m)}$$

$$d\bar{s}_x = dy\,dz\,\hat{a}_x \qquad d\bar{s}_y = dx\,dz\,\hat{a}_y \qquad d\bar{s}_z = dx\,dy\,\hat{a}_z \quad (\text{m}^2)$$

$$ds_x = dy\,dz \qquad ds_y = dx\,dz \qquad ds_z = dx\,dy \quad (\text{m}^2)$$

$$dv = dx\,dy\,dz \quad (\text{m}^3)$$

$$\bar{\nabla}\Phi = \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_y \frac{\partial \Phi}{\partial y} + \hat{a}_z \frac{\partial \Phi}{\partial z} \qquad \left(\frac{\text{units of } \Phi}{\text{m}}\right)$$

$$\bar{\nabla} \bullet \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \left(\frac{\text{units of } A}{\text{m}}\right)$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \text{with} \left(\frac{\text{units of } A}{\text{m}}\right)$$

Cylindrical Coordinates (ρ, ϕ, z)

$$d\bar{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z \qquad \bar{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z \text{ (Note: } \hat{a}_\rho \text{ has } \phi \text{ dependence)}$$

$$d\bar{s}_\rho = \rho\,d\phi\,dz\,\hat{a}_\rho \qquad d\bar{s}_\phi = d\rho\,dz\,\hat{a}_\phi \qquad d\bar{s}_z = \rho\,d\rho\,d\phi\,\hat{a}_z$$

$$ds_\rho = \rho\,d\phi\,dz \qquad ds_\phi = d\rho\,dz \qquad ds_z = \rho\,d\rho\,d\phi$$

$$dv = \rho\,d\rho\,d\phi\,dz$$

$$\bar{\nabla}\Phi = \hat{a}_\rho \frac{\partial \Phi}{\partial \rho} + \hat{a}_\phi \frac{1}{\rho}\frac{\partial \Phi}{\partial \phi} + \hat{a}_z \frac{\partial \Phi}{\partial z}$$

$$\bar{\nabla} \bullet \bar{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_\rho \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{a}_\phi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{a}_z \left(\frac{1}{\rho}\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \phi}\right)$$

Spherical Coordinates (r, θ, ϕ)

$$d\bar{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi \qquad \bar{r} = r\,\hat{a}_r \qquad \text{(Note: } \hat{a}_r \text{ has both } \theta \text{ and } \phi \text{ dependence)}$$

$$d\bar{s}_r = r^2 \sin\theta\,d\theta\,d\phi\,\hat{a}_r \qquad ds_r = r^2 \sin\theta\,d\theta\,d\phi$$

$$d\bar{s}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta \qquad ds_\theta = r\sin\theta\,dr\,d\phi$$

$$d\bar{s}_\phi = r\,dr\,d\theta\,\hat{a}_\phi \qquad ds_\phi = r\,dr\,d\theta$$

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$\bar{\nabla}\Phi = \hat{a}_r \frac{\partial \Phi}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial \Phi}{\partial \theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}$$

$$\bar{\nabla} \bullet \bar{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\,A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

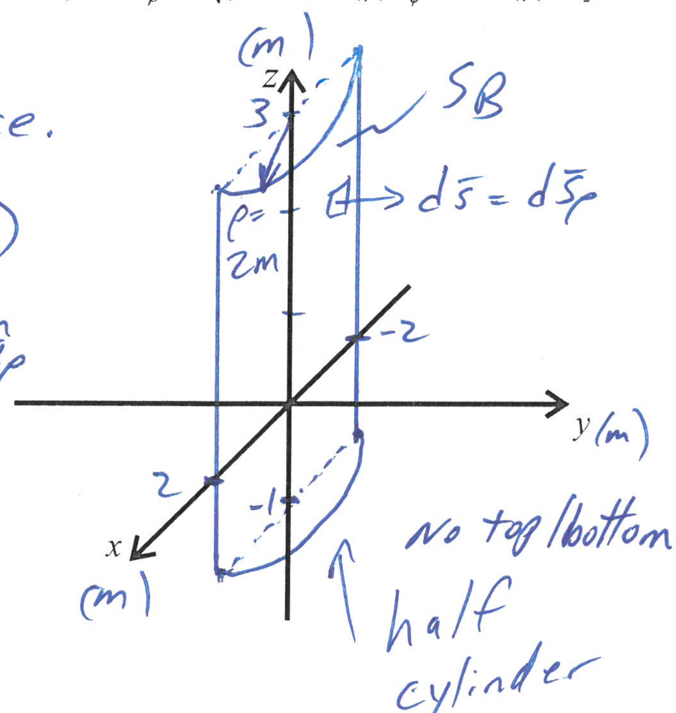
$$\bar{\nabla} \times \bar{A} = \hat{a}_r \left\{\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\,A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\right\} + \hat{a}_\theta \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right] + \hat{a}_\phi \left\{\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\right\}$$

EE 381 Electric and Magnetic Fields Quiz #6 (Fall 2024)

Name Key B

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

On the provided axes, draw a fully labeled sketch of the surface S_B defined by $\rho = 2$ m, $0 \le \phi < \pi$, $-1 \le z \le 3$ m. Next, find the outward (from origin) flux of the vector $\bar{M} = \rho^3 z^2\,\hat{a}_\rho - 9\sqrt{\rho}\,z^2 \tanh(\phi)\,\hat{a}_\phi + 6\sin(\phi) z\,\hat{a}_z$ (mangos/m^2) through S_B.



S_B is NOT a closed surface.

$d\bar{s} = d\bar{s}_\rho = \rho\, d\phi\, dz\, \hat{a}_\rho$ (m^2)

$\bar{M} \cdot d\bar{s}_\rho = (\rho^3 z^2 \hat{a}_\rho \ldots) \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$

$= \rho^4 z^2\, d\phi\, dz$

$$\Psi_{SB} = \iint_{\substack{S_B \\ (\rho = 2m)}} \bar{M} \cdot d\bar{s}_\rho$$

$$= \int_{z=-1}^{3} \int_{\phi=0}^{\pi} (2)^4 z^2\, d\phi\, dz = 16 \int_{z=-1}^{3} z^2\, dz \int_{\phi=0}^{\pi} d\phi$$

$$= 16 \left. \frac{z^3}{3} \right|_{-1}^{3} \left. \phi \right|_0^{\pi} = 16\left[\frac{27}{3} + \frac{1}{3}\right](\pi - 0) = 149.\bar{3}\pi$$

$\Psi_{SB} = 469.1445$ mangos

(Flux) $_{SB}$ = 469.1445 (mangos)

EE 381 Electric and Magnetic Fields optional Quiz #6R (Fall 2024)

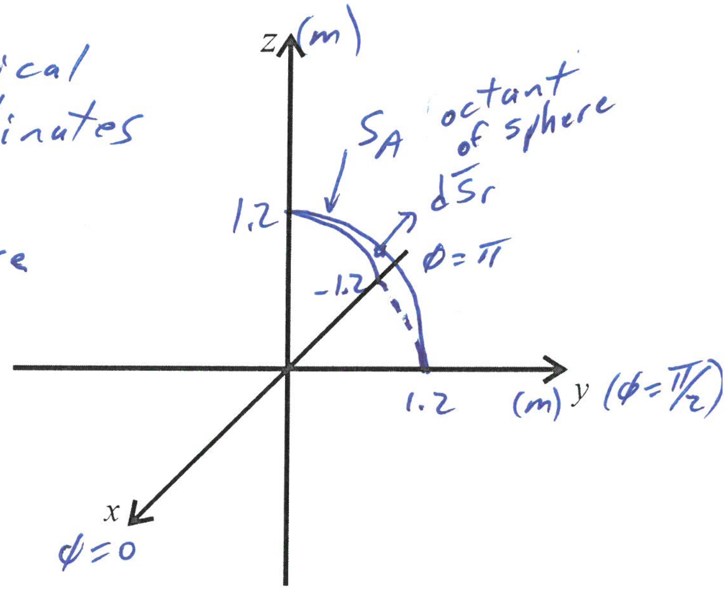
Name Key A

Instructions: (9 points) **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

On the axes shown below, draw a **fully labeled** sketch of the surface S_A defined by $r = 1.2$ m, $0 \le \theta < \pi/2$, and $\pi/2 \le \phi \le \pi$. Next, find the outward (from origin) flux of the vector $\bar{C}$ through surface S_A when $\bar{C} = 2r\,\hat{a}_r + 4r^3\cos(\theta)\,\hat{a}_\theta - 8r^{1/3}\cos(\phi)\,\hat{a}_\phi$ (clams/m²).

$$\Psi_{SA} = \iint_{S_A} \bar{C} \cdot d\bar{s}$$

spherical coordinates



From sketch, the surface differential is:

$$d\bar{s} = d\bar{s}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\bar{C} \cdot d\bar{s} = \bar{C} \cdot d\bar{s}_r$$

$$= (2r\,\hat{a}_r + \ldots) \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= 2r^3 \sin\theta \, d\theta \, d\phi$$

$$\Psi_{SA} = \int_{\phi=\frac{\pi}{2}}^{\pi} \int_{\theta=0}^{\pi/2} 2(1.2)^3 \sin\theta \, d\theta \, d\phi$$

$(r = 1.2\text{m})$

$$= 2(1.2)^3 \; \phi \Big|_{\phi=\pi/2}^{\pi} \; (-\cos\theta)\Big|_{\theta=0}^{\pi/2} = 3.456(\pi - \pi/2)(0+1)$$

$$= 5.428672 \left(\frac{\text{clams}}{\text{m}^2} \cdot \text{m}^2\right)$$

(Flux)$_{SA}$ = 5.42867 (clams)

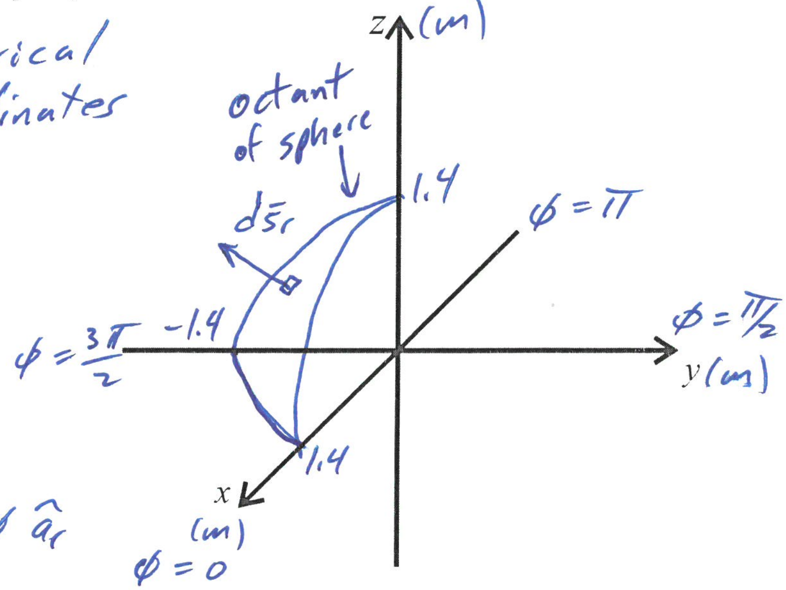
EE 381 Electric and Magnetic Fields optional Quiz #6R (Fall 2024)

Name Key B

Instructions: (9 points) **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

On the axes shown below, draw a **fully labeled** sketch of the surface S_B defined by $r = 1.4$ m, $0 \le \theta < \pi/2$, and $3\pi/2 \le \phi < 2\pi$. Next, find the outward (from origin) flux of the vector $\bar{S}$ through surface S_B when $\bar{S} = 3r\,\hat{a}_r - 6r^3 \sin(\theta)\,\hat{a}_\theta + 9r^{1/4} \sin(\phi)\,\hat{a}_\phi$ (shrimp/m²).

Spherical coordinates



$$\Psi_{SB} = \iint_{S_B} \bar{S} \cdot d\bar{s}$$

From sketch, the surface differential is:

$$d\bar{s} = d\bar{s}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\bar{S} \cdot d\bar{s}_r = (3r\,\hat{a}_r - \ldots) \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= 3r^3 \sin\theta \, d\theta \, d\phi$$

$$\Psi_{SB} = \int_{\phi = 3\pi/2}^{2\pi} \int_{\theta=0}^{\pi/2} 3(1.4)^3 \sin\theta \, d\theta \, d\phi$$

$(r = 1.4\text{m})$

$$= 3(1.4)^3 \, \phi \Big|_{3\pi/2}^{2\pi} (-\cos\theta)\Big|_{\theta=0}^{\pi/2} = 8.232\left(2\pi - \frac{3\pi}{2}\right)(0+1)$$

$$= 12.93079536 \left(\frac{\text{shrimp}}{m^2} \cdot m^2\right)$$

$(\text{Flux})_{SB} =$ 12.9308 (shrimp)