Friday, October 4, 2024 A

EE 381 Electric and Magnetic Fields Quiz #5 (Fall 2024)

Name Key A

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

Given vector $\bar{M} = 2y^2\,\hat{a}_x - 2x\,\hat{a}_y + 4\,\hat{a}_z$, find the Cartesian unit vector in the direction of $\bar{M}$ at the spherical point A(3.2 m, 60°, 140°) as well as point A in Cartesian coordinates.

$\uparrow r \quad \uparrow \theta \quad \uparrow \phi$

Point A in Cartesian coordinates

$x = r\sin\theta\cos\phi = 3.2\sin 60^\circ \cos 140^\circ = -2.122925\text{ m}$

$y = r\sin\theta\sin\phi = 3.2\sin 60^\circ \sin 140^\circ = 1.781345\text{ m}$

$z = r\cos\theta = 3.2\cos 60^\circ = 1.6\text{ m}$

Evaluate $\bar{m}$ @ point $A(-2.123, 1.781, 1.6)$ (m)

$\bar{m} = 2(1.781)^2\hat{a}_x - 2(-2.123)\hat{a}_y + 4\hat{a}_z$

$= 6.346382\,\hat{a}_x + 4.2458493\,\hat{a}_y + 4\,\hat{a}_z$

$\hat{a}_m = \dfrac{\bar{m}}{|\bar{m}|} = \dfrac{6.346382\,\hat{a}_x + 4.2458493\,\hat{a}_y + 4\,\hat{a}_z}{\sqrt{6.3464^2 + 4.24585^2 + 4^2}}$ $\leftarrow 8.62$

$\hat{a}_m = 0.736242\,\hat{a}_x + 0.49256\,\hat{a}_y + 0.464039\,\hat{a}_z$

$A_{cart} =$ (−2.123 m, 1.781 m, 1.6 m)

$\hat{a}_M =$ $0.7362\,\hat{a}_x + 0.4926\,\hat{a}_y + 0.464\,\hat{a}_z$

$\overline{A} \bullet \overline{B} = |\overline{A}||\overline{B}|\cos\theta_{AB}$

$\overline{A} \times \overline{B} = |\overline{A}||\overline{B}|\sin\theta_{AB}\ \hat{a}_n$

Rectangular/Cartesian Coordinates (x, y, z) $\overline{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$

Cylindrical Coordinates (ρ, ϕ, z) $\overline{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z$ (Note: $\hat{a}_\rho$ has ϕ dependence)

Spherical Coordinates (r, θ, ϕ) $\overline{r} = r\,\hat{a}_r$ (Note: $\hat{a}_r$ has both θ and ϕ dependence)

Cartesian Coordinates ⇔ Cylindrical Coordinates

$(x,\ y,\ z)$ ⇔ $(\rho,\ \phi,\ z)$

Point / variable conversions:

$\rho = \sqrt{x^2 + y^2}$ $\quad x = \rho\cos\phi$ $\quad \cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$

$\phi = \tan^{-1}\left(\dfrac{y}{x}\right)$ $\quad y = \rho\sin\phi$ $\quad \sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$

$z = z$ $\quad z = z$

Vector conversions:

$\overline{A} = \hat{a}_\rho A_\rho + \hat{a}_\phi A_\phi + A_z\hat{a}_z = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi$ $\quad \hat{a}_\rho = \cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$

$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi$ $\quad \hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$

$\hat{a}_z = \hat{a}_z$

$A_\rho = A_x\cos\phi + A_y\sin\phi$ $\quad A_x = A_\rho\cos\phi - A_\phi\sin\phi$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$ $\quad A_y = A_\rho\sin\phi + A_\phi\cos\phi$

$A_z = A_z$ $\quad A_z = A_z$

Dot Products:

$\hat{a}_x \bullet \hat{a}_\rho = \cos\phi$ $\quad \hat{a}_y \bullet \hat{a}_\rho = \sin\phi$ $\quad \hat{a}_z \bullet \hat{a}_\rho = 0$

$\hat{a}_x \bullet \hat{a}_\phi = -\sin\phi$ $\quad \hat{a}_y \bullet \hat{a}_\phi = \cos\phi$ $\quad \hat{a}_z \bullet \hat{a}_\phi = 0$

$\hat{a}_x \bullet \hat{a}_z = 0$ $\quad \hat{a}_y \bullet \hat{a}_z = 0$ $\quad \hat{a}_z \bullet \hat{a}_z = 1$

Cartesian Coordinates ⇔ Spherical Coordinates

(x, y, z) ⇔ (r, θ, ϕ)

Point / variable conversions:

$r = \sqrt{x^2+y^2+z^2}$ $\quad x = r\sin\theta\cos\phi$ $\quad \cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$ $\quad \cos\theta = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\quad y = r\sin\theta\sin\phi$

$\theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z}$ $\quad z = r\cos\theta$ $\quad \sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$ $\quad \sin\theta = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$

Vector conversions:

$\overline{A} = \hat{a}_r A_r + A_\theta\hat{a}_\theta + \hat{a}_\phi A_\phi = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$

$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$

$\hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$

$\hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$

$\hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$

$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$

$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$

$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$

$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$

$A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$

$A_z = A_r\cos\theta - A_\theta\sin\theta$

Dot Products:

$\hat{a}_r \bullet \hat{a}_x = \sin\theta\cos\phi$ $\quad \hat{a}_r \bullet \hat{a}_y = \sin\theta\sin\phi$ $\quad \hat{a}_r \bullet \hat{a}_z = \cos\theta$

$\hat{a}_\theta \bullet \hat{a}_x = \cos\theta\cos\phi$ $\quad \hat{a}_\theta \bullet \hat{a}_y = \cos\theta\sin\phi$ $\quad \hat{a}_\theta \bullet \hat{a}_z = -\sin\theta$

$\hat{a}_\phi \bullet \hat{a}_x = -\sin\phi$ $\quad \hat{a}_\phi \bullet \hat{a}_y = \cos\phi$ $\quad \hat{a}_\phi \bullet \hat{a}_z = 0$

EE 381 Electric and Magnetic Fields Quiz #5 (Fall 2024)

Name Key B

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

Given vector $\bar{N} = 3y^2\,\hat{a}_x - 3x\,\hat{a}_y + 3\hat{a}_z$, find the Cartesian unit vector in the direction of $\bar{N}$ at the spherical point B(2.8 m, 110°, 40°) as well as point B in Cartesian coordinates.

$r \quad \theta \quad \phi$

Point B in Cartesian coordinates

$x = r\sin\theta\cos\phi = 2.8\sin 110^\circ\cos 40^\circ = \underline{2.01557\text{ m}}$

$y = r\sin\theta\sin\phi = 2.8\sin 110^\circ\sin 40^\circ = \underline{1.6912638\text{ m}}$

$z = r\cos\theta = 2.8\cos 110^\circ = \underline{-0.9576564\text{ m}}$

Evaluate $\bar{N}$ @ point B (2.016, 1.691, −0.958) (m)

$\bar{N} = 3(1.6913)^2\hat{a}_x - 3(2.0156)\hat{a}_y + 3\hat{a}_z$

$= 8.58112\,\hat{a}_x - 6.04671\,\hat{a}_y + 3\hat{a}_z$

$\hat{a}_N = \dfrac{\bar{N}}{|\bar{N}|} = \dfrac{8.58112\,\hat{a}_x - 6.04671\,\hat{a}_y + 3\hat{a}_z}{\sqrt{8.58112^2 + (-6.04671)^2 + 3^2}} \leftarrow 10.9178$

$\underline{\hat{a}_N = 0.785975\,\hat{a}_x - 0.55384\,\hat{a}_y + 0.27478\,\hat{a}_z}$

$B_{\text{cart}} =$ (2.0156 m, 1.6913 m, −0.9577 m)

$\hat{a}_N =$ $0.785975\,\hat{a}_x - 0.55384\,\hat{a}_y + 0.27478\,\hat{a}_z$

$$\overline{A}\bullet\overline{B} = |\overline{A}||\overline{B}|\cos\theta_{AB}$$

$$\overline{A}\times\overline{B} = |\overline{A}||\overline{B}|\sin\theta_{AB}\,\hat{a}_n$$

Rectangular/Cartesian Coordinates (x, y, z) $\quad \overline{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$

Cylindrical Coordinates (ρ, ϕ, z) $\quad \overline{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z$ (Note: $\hat{a}_\rho$ has ϕ dependence)

Spherical Coordinates (r, θ, ϕ) $\quad \overline{r} = r\,\hat{a}_r$ (Note: $\hat{a}_r$ has both θ and ϕ dependence)

Cartesian Coordinates $\Leftrightarrow$ Cylindrical Coordinates

$(x, y, z) \Leftrightarrow (\rho, \phi, z)$

Point / variable conversions:

$\rho = \sqrt{x^2 + y^2}$ $\qquad x = \rho\cos\phi$ $\qquad \cos\phi = \dfrac{x}{\sqrt{x^2 + y^2}}$

$\phi = \tan^{-1}\left(\dfrac{y}{x}\right)$ $\qquad y = \rho\sin\phi$ $\qquad \sin\phi = \dfrac{y}{\sqrt{x^2 + y^2}}$

$z = z$ $\qquad z = z$

Vector conversions:

$\overline{A} = \hat{a}_\rho A_\rho + \hat{a}_\phi A_\phi + A_z\hat{a}_z = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi$ $\qquad \hat{a}_\rho = \cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$

$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi$ $\qquad \hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$

$\hat{a}_z = \hat{a}_z$

$A_\rho = A_x\cos\phi + A_y\sin\phi$ $\qquad A_x = A_\rho\cos\phi - A_\phi\sin\phi$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$ $\qquad A_y = A_\rho\sin\phi + A_\phi\cos\phi$

$A_z = A_z$ $\qquad A_z = A_z$

Dot Products:

$\hat{a}_x\bullet\hat{a}_\rho = \cos\phi$ $\qquad \hat{a}_y\bullet\hat{a}_\rho = \sin\phi$ $\qquad \hat{a}_z\bullet\hat{a}_\rho = 0$

$\hat{a}_x\bullet\hat{a}_\phi = -\sin\phi$ $\qquad \hat{a}_y\bullet\hat{a}_\phi = \cos\phi$ $\qquad \hat{a}_z\bullet\hat{a}_\phi = 0$

$\hat{a}_x\bullet\hat{a}_z = 0$ $\qquad \hat{a}_y\bullet\hat{a}_z = 0$ $\qquad \hat{a}_z\bullet\hat{a}_z = 1$

Cartesian Coordinates $\Leftrightarrow$ Spherical Coordinates

$(x, y, z) \Leftrightarrow (r, \theta, \phi)$

Point / variable conversions:

$r = \sqrt{x^2 + y^2 + z^2}$ $\qquad x = r\sin\theta\cos\phi$ $\qquad \cos\phi = \dfrac{x}{\sqrt{x^2 + y^2}}$ $\qquad \cos\theta = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad y = r\sin\theta\sin\phi$

$\theta = \tan^{-1}\dfrac{\sqrt{x^2 + y^2}}{z}$ $\qquad z = r\cos\theta$ $\qquad \sin\phi = \dfrac{y}{\sqrt{x^2 + y^2}}$ $\qquad \sin\theta = \dfrac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$

Vector conversions:

$\overline{A} = \hat{a}_r A_r + A_\theta\hat{a}_\theta + \hat{a}_\phi A_\phi = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$

$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$

$\hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$

$\hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$

$\hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$

$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$

$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$

$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$

$A_\phi = -A_x\sin\phi + A_y\cos\phi$

$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$

$A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$

$A_z = A_r\cos\theta - A_\theta\sin\theta$

Dot Products:

$\hat{a}_r\bullet\hat{a}_x = \sin\theta\cos\phi$ $\qquad \hat{a}_r\bullet\hat{a}_y = \sin\theta\sin\phi$ $\qquad \hat{a}_r\bullet\hat{a}_z = \cos\theta$

$\hat{a}_\theta\bullet\hat{a}_x = \cos\theta\cos\phi$ $\qquad \hat{a}_\theta\bullet\hat{a}_y = \cos\theta\sin\phi$ $\qquad \hat{a}_\theta\bullet\hat{a}_z = -\sin\theta$

$\hat{a}_\phi\bullet\hat{a}_x = -\sin\phi$ $\qquad \hat{a}_\phi\bullet\hat{a}_y = \cos\phi$ $\qquad \hat{a}_\phi\bullet\hat{a}_z = 0$