EE 381 Electric and Magnetic Fields Quiz #4 (Fall 2024)

Name Key A

Instructions: Closed book & notes. Place answers in indicated spaces and show all work for credit.

Useful equations: $\bar{A}\cdot\bar{B}=|\bar{A}||\bar{B}|\cos\theta_{AB}$, $\bar{A}\times\bar{B}=|\bar{A}||\bar{B}|\sin\theta_{AB}\,\hat{a}_n$, $\bar{r}_{\text{position}}=x\hat{a}_x+y\hat{a}_y+z\hat{a}_z$

Given: Cartesian points A(16,7,1), B(7,8,-9), C(-3,-9,-4) & D(4,-11,-3) [units of meters]

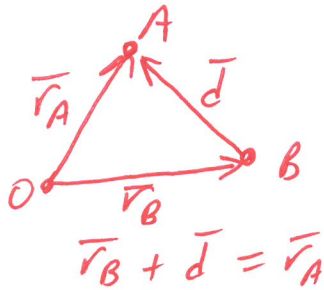
Vectors $\bar{M}=22\hat{a}_x-9\hat{a}_y+13\hat{a}_z$, $\bar{L}=18\hat{a}_x+16\hat{a}_y+12\hat{a}_z$, and $\bar{N}=-10\hat{a}_x-7\hat{a}_y-14\hat{a}_z$

Find:

a) The position vectors ($\bar{r}_A$ and $\bar{r}_B$) for points A and B.

$\bar{r}_A =$ $16\hat{a}_x+7\hat{a}_y+\hat{a}_z$ (m) $\quad \bar{r}_B =$ $7\hat{a}_x+8\hat{a}_y-9\hat{a}_z$ (m)

b) The distance vector $\bar{d}$ from point B to point A.



$\bar{r}_B+\bar{d}=\bar{r}_A$

$\bar{d}=\bar{r}_A-\bar{r}_B$

$=(16-7)\hat{a}_x+(7-8)\hat{a}_y+(1-(-9))\hat{a}_z$

$\bar{d} =$ $9\hat{a}_x-\hat{a}_y+10\hat{a}_z$ (m)

c) The unit vector in the direction of $\bar{N}$.

$\hat{a}_N=\dfrac{\bar{N}}{|\bar{N}|}$

$|\bar{N}|=\sqrt{\bar{N}\cdot\bar{N}}=\sqrt{(-10)^2+(-7)^2+(-14)^2}=\sqrt{345}$

$\hat{a}_N=\dfrac{-10\hat{a}_x-7\hat{a}_y-14\hat{a}_z}{\sqrt{345}}$

$\hat{a}_N =$ $-0.538\hat{a}_x-0.377\hat{a}_y-0.754\hat{a}_z$

d) The smallest angle ϕ (in degrees) that $\bar{N}$ makes with the positive x-axis ($\hat{a}_x$)

$\bar{N}\cdot\hat{a}_x=|\bar{N}|(1)\cos\phi \Rightarrow \phi=\cos^{-1}\left(\dfrac{\bar{N}\cdot\hat{a}_x}{|\bar{N}|}\right)$

$|\bar{N}|=\sqrt{345}$

$\bar{N}\cdot\hat{a}_x=(-10\hat{a}_x-7\hat{a}_y-14\hat{a}_z)\cdot\hat{a}_x$

$=-10$

$=\cos^{-1}\left(\dfrac{-10}{\sqrt{345}}\right)$

$\phi =$ 122.5736°

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Name Key B

Instructions: Closed book & notes. Place answers in indicated spaces and show all work for credit.

Useful equations: $\bar{A}\cdot\bar{B}=|\bar{A}||\bar{B}|\cos\theta_{AB}$, $\bar{A}\times\bar{B}=|\bar{A}||\bar{B}|\sin\theta_{AB}\,\hat{a}_n$, $\bar{r}_{\text{position}}=x\hat{a}_x+y\hat{a}_y+z\hat{a}_z$

Given: Cartesian points A(16,7,1), B(7,8,-9), C(-3,-9,-4) & D(4,-11,-3) [units of meters]

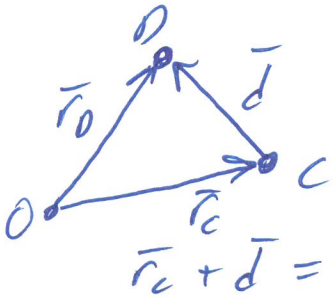
Vectors $\bar{M}=22\hat{a}_x-9\hat{a}_y+13\hat{a}_z$, $\bar{L}=18\hat{a}_x+16\hat{a}_y+12\hat{a}_z$, and $\bar{N}=-10\hat{a}_x-7\hat{a}_y-14\hat{a}_z$

Find:

a) The position vectors ($\bar{r}_C$ and $\bar{r}_D$) for points C and D.

$\bar{r}_C = -3\hat{a}_x-9\hat{a}_y-4\hat{a}_z$ (m) $\quad \bar{r}_D = 4\hat{a}_x-11\hat{a}_y-3\hat{a}_z$ (m)

b) The distance vector $\bar{d}$ from point C to point D.



$\bar{r}_C+\bar{d}=\bar{r}_D$

$\bar{d}=\bar{r}_D-\bar{r}_C$

$=(4-(-3))\hat{a}_x+(-11-(-9))\hat{a}_y+(-3-(-4))\hat{a}_z$

$\bar{d} = 7\hat{a}_x-2\hat{a}_y+\hat{a}_z$ (m)

c) The unit vector in the direction of $\bar{L}$.

$\hat{a}_L=\dfrac{\bar{L}}{|\bar{L}|}$ $\quad |\bar{L}|=\sqrt{\bar{L}\cdot\bar{L}}=\sqrt{18^2+16^2+12^2}=\sqrt{724}$

$=\dfrac{18\hat{a}_x+16\hat{a}_y+12\hat{a}_z}{\sqrt{724}}$

$\hat{a}_L = 0.669\hat{a}_x+0.595\hat{a}_y+0.446\hat{a}_z$

d) The smallest angle θ (in degrees) that $\bar{L}$ makes with the positive z-axis ($\hat{a}_z$)

$\bar{L}\cdot\hat{a}_z=|\bar{L}|(1)\cos\theta \rightarrow \theta=\cos^{-1}\left(\dfrac{\bar{L}\cdot\hat{a}_z}{|\bar{L}|}\right)$

$|\bar{L}|=\sqrt{724}$ $\quad =\cos^{-1}\left(\dfrac{12}{\sqrt{724}}\right)$

$\bar{L}\cdot\hat{a}_z=(18\hat{a}_x+16\hat{a}_y+12\hat{a}_z)\cdot\hat{a}_z$

$=12$

$\theta = 63.5142^\circ$