Monday, September 9, 2024 A

EE 381 Electric and Magnetic Fields Quiz #1 (Fall 2024)

Name Key A

Instructions: Closed book & notes. Place answers in indicated spaces and show all work for credit.

Gru is developing a line of self-heating two-wire transmission lines with thorium (ε_0, μ_0, 6.37×10^6 S/m) conductors of 2 mm diameter separated by 11 mm by an insulator ($1.8\varepsilon_0$, μ_0, 2×10^{-4} S/m) for artic operations. Determine the conductor skin depth and per-unit-length parameters at 440 MHz.

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi\, 440\times10^6 (4\pi\times10^{-7}) 6.37\times10^6}}$$

$$= 9.506575\times10^{-6}\ \text{m} = \underline{9.50657\ \mu\text{m}}$$

$$R = \frac{1}{\pi a \delta \sigma_c} = \frac{1}{\pi\left(\frac{2\times10^{-3}}{2}\right) 9.5066\times10^{-6}(6.37\times10^6)} = \underline{5.25638\ \Omega/\text{m}}$$

$$L = \frac{\mu}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) = \frac{4\pi\times10^{-7}}{\pi}\cosh^{-1}\left(\frac{11}{2}\right) = \underline{9.558106\times10^{-7}\ \text{H/m}}$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(d/2a)} = \frac{\pi(2\times10^{-4})}{\cosh^{-1}(11/2)} = \underline{0.000262947}\ \text{S/m}$$

$$C = \frac{\pi\epsilon}{\cosh^{-1}(d/2a)} = \frac{\pi(1.8)8.8541878\times10^{-12}}{\cosh^{-1}(11/2)}$$

$$= \underline{2.09536\times10^{-11}\ \text{F/m}}$$

Skin depth = 9.50657 μm R = 5.25638 Ω/m

L = 955.811 nH/m G = 262.947 μS/m C = 20.9536 pF/m

Useful equations: $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$, $\lambda = \frac{u}{f} = \frac{2\pi}{\beta}$,

$Z_0 = V_0^+ / I_0^+ = \sqrt{(R + j\omega L)/(G + j\omega C)}$

TABLE 11.1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ ($\delta \ll a, c - b$)	$\frac{1}{\pi a\delta\sigma_c}$ ($\delta \ll a$)	$\frac{2}{w\delta\sigma_c}$ ($\delta \ll t$)
L (H/m)	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
G (S/m)	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
C (F/m)	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ ($w \gg d$)

$*\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\frac{d}{2a} \simeq \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

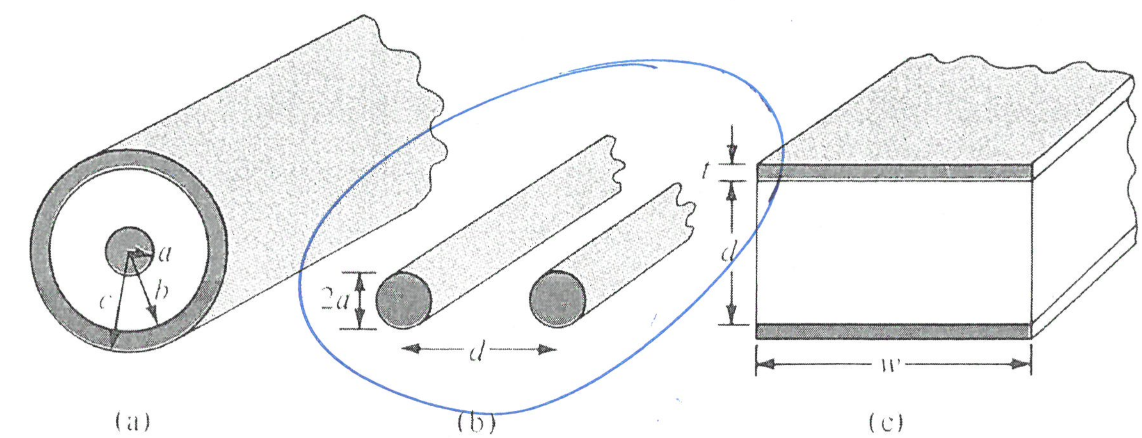

Figure 11.2 Common transmission lines: (**a**) coaxial line, (**b**) two-wire line, (**c**) planar line.

EE 381 Electric and Magnetic Fields Quiz #1 (Fall 2024)

Name Key B

Instructions: Closed book & notes. Place answers in indicated spaces and show all work for credit.

Dr. Nefario is developing a unique self-illuminating two-wire transmission line with pure radium (ε_0, μ_0, 1×10^6 S/m) conductors of 3 mm diameter separated by 13 mm by an insulator ($1.6\varepsilon_0$, μ_0, 6×10^{-5} S/m) for night operations. Determine the conductor skin depth and per-unit-length parameters at 555 MHz.

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi\, 555\times10^6 (4\pi\times10^{-7})(1\times10^6)}}$$

$$= 2.13636\times10^{-5} = \underline{21.3636\ \mu m}$$

$$R = \frac{1}{\pi a \delta \sigma_c} = \frac{1}{\pi\left(\frac{3\times10^{-3}}{2}\right)2.13636\times10^{-5}(10^6)} = \underline{9.93311\ \Omega/m}$$

$$L = \frac{\mu}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) = \frac{4\pi\times10^{-7}}{\pi}\cosh^{-1}\left(13/3\right)$$

$$= 8.58359\times10^{-7}\ H/m = \underline{858.359\ nH/m}$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(d/2a)} = \frac{\pi\, 6\times10^{-5}}{\cosh^{-1}(13/3)} = \underline{8.784\times10^{-5}\ S/m}$$

$$C = \frac{\pi\epsilon}{\cosh^{-1}(d/2a)} = \frac{\pi(1.6)8.8541878\times10^{-12}}{\cosh^{-1}(13/3)}$$

$$= \underline{2.074\times10^{-11}\ F/m}$$

Skin depth = 21.3636 μm R = 9.93311 Ω/m

L = 858.359 nH/m G = 87.84 μS/m C = 20.74 pF/m

Useful equations: $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$, $\lambda = \dfrac{u}{f} = \dfrac{2\pi}{\beta}$,

$Z_0 = V_0^+ / I_0^+ = \sqrt{(R + j\omega L)/(G + j\omega C)}$

TABLE 11.1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\dfrac{1}{2\pi\delta\sigma_c}\left[\dfrac{1}{a} + \dfrac{1}{b}\right]$ $(\delta \ll a, c - b)$	$\dfrac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\dfrac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
L (H/m)	$\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$	$\dfrac{\mu}{\pi}\cosh^{-1}\dfrac{d}{2a}$	$\dfrac{\mu d}{w}$
G (S/m)	$\dfrac{2\pi\sigma}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\sigma}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\sigma w}{d}$
C (F/m)	$\dfrac{2\pi\varepsilon}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\varepsilon}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\varepsilon w}{d}$ $(w \gg d)$

$*\delta = \dfrac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\dfrac{d}{2a} \simeq \ln\dfrac{d}{a}$ if $\left[\dfrac{d}{2a}\right]^2 \gg 1$.

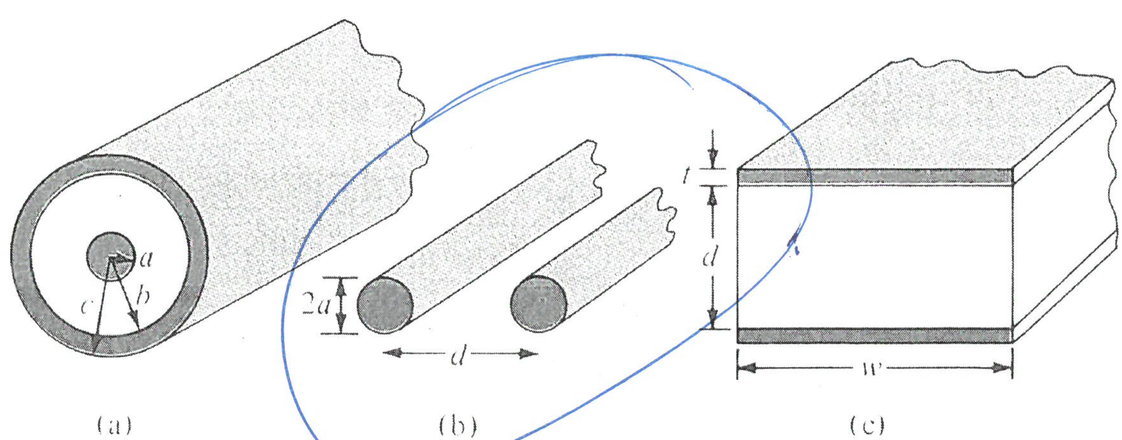

Figure 11.2 Common transmission lines: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line.