EE 381 Electric & Magnetic Fields Examination #1 (Fall 2024)

Name Key B

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheets in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) Given points $B(2, -7, 3)$, $A(5, 8, -3)$, and $D(3, -4, 8)$ (in meters), and vectors, $\bar{F} = xz\,\hat{a}_x - 3x^3\,\hat{a}_y + 2y\,\hat{a}_z$ (N), $\bar{O} = 2\,\hat{a}_x - 4\,\hat{a}_z$ (N), and $\bar{G} = 9\,\hat{a}_x - 2\,\hat{a}_y + 7\,\hat{a}_z$ (m) find:

a) the vector product of $\bar{O}$ with the position vector pointing to point B,

$$\bar{r}_B = 2\hat{a}_x - 7\hat{a}_y + 3\hat{a}_z \text{ (m)}$$

$$\underset{(N)}{\bar{O}} \times \underset{(m)}{\bar{r}_B} = \left| \begin{matrix} \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x & \hat{a}_y \\ 2 & 0 & -4 & 2 & 0 \\ 2 & -7 & 3 & 2 & -7 \end{matrix} \right| = \hat{a}_x[0-(-4)(-7)] + \hat{a}_y[-4(2)-2(3)] + \hat{a}_z[2(-7)-0] \text{ N}\cdot\text{m}$$

$$\bar{O}\times\bar{r}_B = -28\,\hat{a}_x - 14\,\hat{a}_y - 14\,\hat{a}_z \text{ (N}\cdot\text{m)}$$

b) smallest angle (in degrees) that the distance vector from point A to B makes with $\bar{G}$,

$$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (2\hat{a}_x - 7\hat{a}_y + 3\hat{a}_z) - (5\hat{a}_x + 8\hat{a}_y - 3\hat{a}_z)$$

$$= -3\,\hat{a}_x - 15\,\hat{a}_y + 6\,\hat{a}_z \text{ (m)} \Rightarrow r_{AB} = \sqrt{(-3)^2+(-15)^2+6^2} = \sqrt{270}$$

$$\bar{G} = 9\hat{a}_x - 2\hat{a}_y + 7\hat{a}_z \Rightarrow |\bar{G}| = \sqrt{9^2+(-2)^2+7^2} = \sqrt{134}$$

$$\bar{r}_{AB}\cdot\bar{G} = -3(9) + (-15)(-2) + 6(7) = 45 = |\bar{r}_{AB}||\bar{G}|\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{45}{\sqrt{270}\sqrt{134}}\right) \longrightarrow \theta = 76.315^\circ$$

c) the unit vector for vector field $\bar{F}$ evaluated at point D, and $(3, -4, 8)$

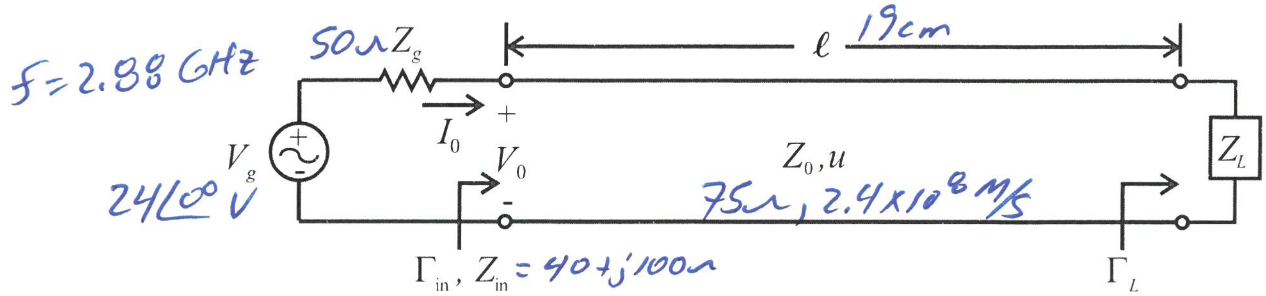
$$\bar{F}_D = 3(8)\hat{a}_x - 3(3)^3\hat{a}_y + 2(-4)\hat{a}_z = 24\,\hat{a}_x - 81\,\hat{a}_y - 8\hat{a}_z$$

$$|\bar{F}_D| = \sqrt{24^2 + (-81)^2 + (-8)^2} = \sqrt{7201}$$

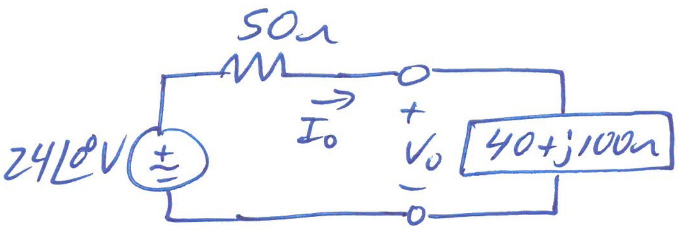
$$\hat{a}_{FD} = \frac{\bar{F}_D}{|\bar{F}_D|} = \frac{24\,\hat{a}_x - 81\,\hat{a}_y - 8\,\hat{a}_z}{\sqrt{7201}}$$

$$\hat{a}_{FD} = 0.2828\,\hat{a}_y - 0.9545\,\hat{a}_y - 0.0943\,\hat{a}_z$$

2) For the lossless transmission circuit shown, $Z_0 = 75\ \Omega$, $u = 2.4 \times 10^8$ m/s, $\ell = 19$ cm, $f = 2.88$ GHz, $Z_{in} = 40 + j\,100\ \Omega$, $Z_g = 50\ \Omega$, and $V_g = 24\ \angle 0^\circ$ (V). **Calculate** V_0, I_0, Γ_{in}, VSWR, V_0^+ and the time-average power to the load P_L. **Express complex answers in phasor form A∠θ with angle in degrees.**



Equiv. Input Circuit



$$I_0 = \frac{24\angle 0^\circ}{50 + (40 + j100)} = 0.17839\angle -48.01279^\circ \text{ A}$$

$$V_0 = I_0(40 + j100) = 19.213255\angle 20.1858^\circ \text{ V}$$

$$V_0^+ = \tfrac{1}{2}\left[V_0 + I_0 Z_0\right] = \tfrac{1}{2}\left[19.2\angle 20.19^\circ + (0.178\angle -48^\circ)75\right] = 13.5931\angle -7.00360^\circ \text{ V}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{(40 + j100) - 75}{(40 + j100) + 75} = 0.69521\angle 68.28^\circ$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.6952}{1 - 0.6952} = 5.562$$

Lossless TL

$$P_{in} = P_L = \tfrac{1}{2}\,\mathrm{Re}\{V_0 I_0^*\} = \tfrac{1}{2}\,\mathrm{Re}\{(19.21\angle 20.2^\circ)(0.1784\angle +48.01^\circ)\} = 0.6365 \text{ W}$$

V_0 = 19.213∠20.186° V $\quad I_0$ = 0.1784∠−48.013° A $\quad$ VSWR = 5.562

Γ_{in} = 0.6952∠68.281° $\quad V_0^+$ = 13.593∠−7.004° V $\quad P_L$ = 0.6365 W

3) Given the cylindrical points $E(3, \pi/3, -2)$, $F(5, 4\pi/3, -3)$, and $G(6, 2\pi/3, 8)$ (distances in units of meters), and vectors $\overline{P} = -\rho^2\,\hat{a}_\rho + 3z\,\hat{a}_\phi - 7\sin(\phi)z\,\hat{a}_z$ and $\overline{R} = 2\,\hat{a}_\rho - 4\,\hat{a}_\phi + 8\,\hat{a}_z$, find:

a) the Cartesian coordinates for points E and G,

Point $E(3, \pi/3, -2)$

$x = \rho\cos\phi = 3\cos\frac{\pi}{3} = 1.5\,m$

$y = \rho\sin\phi = 3\sin\frac{\pi}{3} = 2.598\,m$

$z = -2\,m$

pt $G\ (6, 2\pi/3, 8)$

$x = 6\cos(2\pi/3) = -3\,m$

$y = 6\sin(2\pi/3) = 5.196\,m$

$z = 8\,m$

$E_{cart}(1.5, 2.598, -2)\ (m)$ & $G_{cart}(-3, 5.196, 8)\ (m)$

b) the vector component of $\overline{P}$ parallel to $\overline{R}$ with both vectors at point F, and $(5, 4\pi/3, -3)$

$$\hat{a}_R = \frac{\overline{R}}{|\overline{R}|} = \frac{2\hat{a}_\rho - 4\hat{a}_\phi + 8\hat{a}_z}{\sqrt{2^2+(-4)^2+8^2}} = \frac{2\hat{a}_\rho - 4\hat{a}_\phi + 8\hat{a}_z}{\sqrt{84}}$$

$$\overline{P}_F = -(5)^2\hat{a}_\rho + 3(-3)\hat{a}_z - 7\sin(4\pi/3)(-3)\hat{a}_z$$

$$= -25\hat{a}_\rho - 9\hat{a}_z - 18.1865335\,\hat{a}_z$$

$$\overline{P}_{AR} = (\overline{P}_F \cdot \hat{a}_R)\,\hat{a}_R$$

$$= \left(\frac{-25(2) - 9(-4) - 18.1865(8)}{\sqrt{\ }}\right)\left(\frac{2\hat{a}_\rho - 4\hat{a}_\phi + 8\hat{a}_z}{\sqrt{84}}\right)$$

$$= \frac{-159.49227}{84}(2\hat{a}_\rho - 4\hat{a}_\phi + 8\hat{a}_z)$$

$$\overline{P}_{aR} = -3.7974\,\hat{a}_\rho + 7.5949\,\hat{a}_\phi - 15.1897\,\hat{a}_z$$

c) the distance between points E and G.

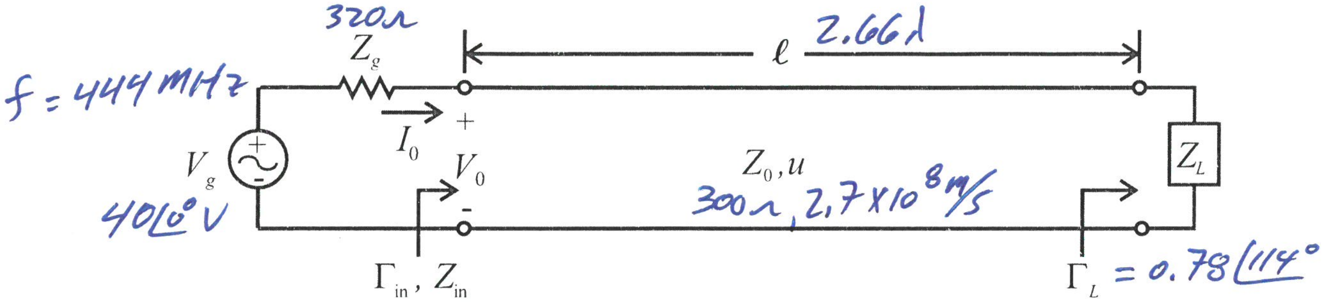
$$\overline{r}_{EG} = \overline{r}_G - \overline{r}_E = (1.5+3)\hat{a}_x + (2.598 - 5.196)\hat{a}_y + (-2-8)\hat{a}_z$$

$$D_{EG} = \sqrt{\overline{r}_{EG}\cdot\overline{r}_{EG}} = \sqrt{4.5^2 + (-2.598)^2 + (-10)^2}$$

$$= \sqrt{127} = 11.26943\,m$$

$$D_{EG} = 11.269\ m$$

4) For the lossless transmission circuit shown, $Z_0 = 300\ \Omega$, $u = 2.7 \times 10^8$ m/s, $\ell = 2.66\lambda$, $f = 444$ MHz, $\Gamma_L = 0.78\angle 114^\circ$, $Z_g = 320\ \Omega$, and $V_g = 40\ \angle 0^\circ$ (V). Using a **Smith chart (show & label all work)**, find Z_L, Γ_{in}, Z_{in}, VSWR, and Y_{in}. **Express reflection coefficient answers in phasor form A∠θ w/ angle in degrees & admittance/impedance in rectangular format a + *j*b.**



1) Plot $\Gamma_L = 0.78\angle 114^\circ$ on Smith chart by drawing radial line from center thru 114° & a circle of radius $|\Gamma| = 0.78$ (use scale @ bottom). Also mark SWR (VSWR) scale.

2) Read $\underline{VSWR = 8.1}$. Read $z_L = 0.175 + j0.64\ \Omega/\Omega$.
Calculate $Z_L = z_L Z_0 = (0.175 + j0.64)300 = \underline{52.5 + j192\ \Omega}$

3) Move $2.66\lambda \to 0.16\lambda$ 'WAVELENGTHS TOWARD GENERATOR' (i.e., $0.0918 + 0.16 = 0.2518$) on circle of $|\Gamma| = 0.78$ to z_{in} / Γ_{in} point. and draw radial line.

4) Read $\underline{\angle\Gamma_{in} = -1.2^\circ}$ and $z_{in} = 8 - j0.67\ \Omega/\Omega$.
$Z_{in} = z_{in} Z_0 = (8 - j0.67)300 = \underline{2400 - j201\ \Omega}$

5) Go 180° around $|\Gamma| = 0.78$ circle from z_{in} to $y_{in} = 0.124 + j0.01\ S/S$. $Y_{in} = \dfrac{y_{in}}{Z_0}$

$$Y_{in} = \frac{0.124 + j0.01}{300} = \underline{0.00041 + j0.0000335\ S}$$

Z_L = 52.5 + j192 Ω Γ_{in} = 0.78∠−1.2° VSWR = 8.1

Z_{in} = 2400 − j201 Ω Y_{in} = 0.00041 + j0.0000335
= 0.413 + j0.03 mS

KEY B

Simple Smith Chart

$Z_0 = 300\ \Omega$

$f = 444$ MHZ

$u = 2.7*10^8$ m/s