EE 381 Electric & Magnetic Fields Examination #1 (Fall 2024)

Name Key A

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheets in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) Given points $B(2, -7, 3)$, $A(5, 8, -3)$, and $D(3, -4, 8)$ (in meters), and vectors, $\bar{F} = xz\,\hat{a}_x - 3x^3\,\hat{a}_y + 2y\,\hat{a}_z$ (N), $\bar{O} = 2\,\hat{a}_x - 4\,\hat{a}_z$ (N), and $\bar{G} = 9\,\hat{a}_x - 2\,\hat{a}_y + 7\,\hat{a}_z$ (m) find:

a) the vector product of $\bar{O}$ with the position vector pointing to point A,

$\bar{r}_A = 5\hat{a}_x + 8\hat{a}_y - 3\hat{a}_z$ (m)

$$\underset{(N)}{\bar{O}} \times \underset{(m)}{\bar{r}_A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & 0 & -4 \\ 5 & 8 & -3 \end{vmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ 2 & 0 \\ 5 & 8 \end{matrix} = \hat{a}_x[0-(-4)8] + \hat{a}_y[-4(5)-2(-3)] + \hat{a}_z[2(8)-0] \text{ N·m}$$

$\bar{O}\times\bar{r}_A = 32\,\hat{a}_x - 14\,\hat{a}_y + 16\,\hat{a}_z$ (N·m)

b) smallest angle (in degrees) that the distance vector from point B to A makes with $\bar{G}$,

$\bar{r}_{BA} = \bar{r}_A - \bar{r}_B = (5\hat{a}_x + 8\hat{a}_y - 3\hat{a}_z) - (2\hat{a}_x - 7\hat{a}_y + 3\hat{a}_z)$

$= 3\hat{a}_x + 15\hat{a}_y - 6\hat{a}_z$ (m) $\Rightarrow |\bar{r}_{BA}| = \sqrt{3^2+15^2+(-6)^2} = \sqrt{270}$

$\bar{G} = 9\hat{a}_x - 2\hat{a}_y + 7\hat{a}_z \Rightarrow |\bar{G}| = \sqrt{9^2+(-2)^2+7^2} = \sqrt{134}$

$\bar{r}_{BA}\cdot\bar{G} = 3(9) + 15(-2) + (-6)7 = -45 = |\bar{r}_{BA}||\bar{G}|\cos\theta$

$\theta = \cos^{-1}\left(\frac{-45}{\sqrt{270}\sqrt{134}}\right) \longrightarrow$

$\theta = 103.685^\circ$

c) the unit vector for vector field $\bar{F}$ evaluated at point D, and $D(3,-4,8)$

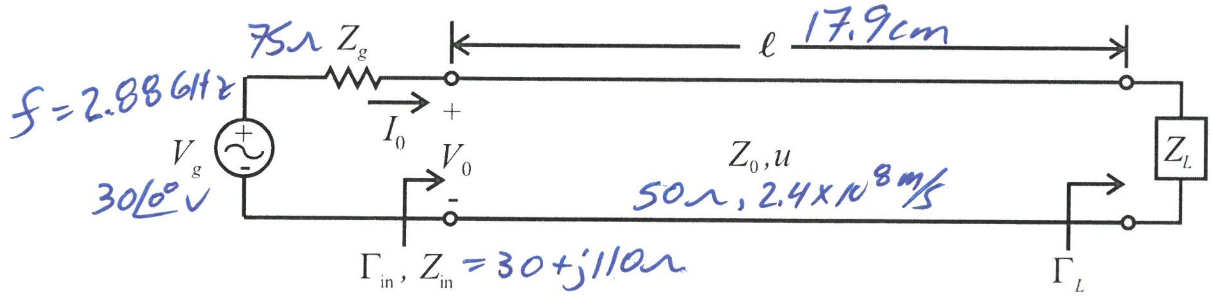
$\bar{F}_D = 3(8)\hat{a}_x - 3(3)^3\hat{a}_y + 2(-4)\hat{a}_z = 24\hat{a}_x - 81\hat{a}_y - 8\hat{a}_z$

$|\bar{F}_D| = \sqrt{24^2 + (-81)^2 + (-8)^2} = \sqrt{7201}$

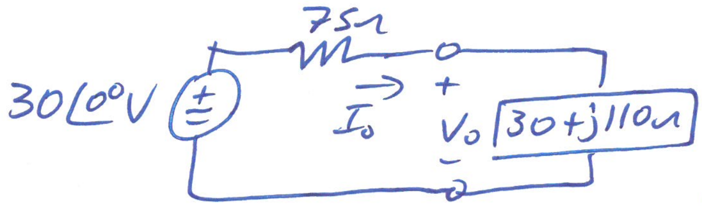
$\hat{a}_{FD} = \dfrac{24\hat{a}_x - 81\hat{a}_y - 8\hat{a}_z}{\sqrt{7201}}$

$\hat{a}_{F_D} = 0.2828\hat{a}_x - 0.9545\hat{a}_y - 0.0943\hat{a}_z$

2) For the lossless transmission circuit shown, $Z_0 = 50\ \Omega$, $u = 2.4 \times 10^8$ m/s, $\ell = 17.9$ cm, $f = 2.88$ GHz, $Z_{in} = 30 + j\,110\ \Omega$, $Z_g = 75\ \Omega$, and $V_g = 30\ \angle 0^\circ$ (V). **Calculate** V_0, I_0, Γ_{in}, VSWR, V_0^+ and the time-average power to the load P_L. **Express complex answers in phasor form A∠θ with angle in degrees.**



Equiv. Input Circuit



$$I_o = \frac{30\angle 0^\circ}{75 + (30 + j110)} = 0.19728\angle -46.3322^\circ\ A$$

$$V_o = I_o(30 + j110) = 22.49324\angle 28.41266^\circ\ V$$

$$V_o^+ = \tfrac{1}{2}\{V_o + I_o Z_o\} = \tfrac{1}{2}\{22.5\angle 28.4^\circ + 0.1973\angle -46.3^\circ(50)\} = \underline{13.416\angle 7.64^\circ\ V}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{(30 + j110) - 50}{(30 + j110) + 50} = 0.821995\angle 46.3322^\circ$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.822}{1 - 0.822} = \underline{10.236}$$

Lossless TL $P_{in} = P_L = \tfrac{1}{2}\,\mathrm{Re}\{V_o I_o^*\}$

$$= \tfrac{1}{2}\,\mathrm{Re}\{22.5\angle 28.4^\circ(0.1973\angle +46.3322^\circ)\}$$

$$= \underline{0.5838\ W}$$

V_0 = 22.493∠28.413° V I_0 = 0.1973∠-46.332° A VSWR = 10.236

Γ_{in} = 0.822∠46.332° V_0^+ = 13.416∠7.64° V P_L = 0.5838 W

$\rho \quad \phi \quad z$

3) Given the cylindrical points $E(2, \pi/3, -3)$, $F(5, 5\pi/3, -3)$, and $G(6, 2\pi/3, 8)$ (distances in units of meters), and vectors $\overline{P} = -\rho^2\,\hat{a}_\rho + 3z\,\hat{a}_\phi - 7\sin(\phi)z\,\hat{a}_z$ and $\overline{R} = 3\,\hat{a}_\rho - 6\,\hat{a}_\phi + 8\,\hat{a}_z$, find:

a) the Cartesian coordinates for points F and G,

Point $F(5, 5\pi/3, -3)$

$x = \rho\cos\phi = 5\cos(5\pi/3) = 2.5\ m$

$y = \rho\sin\phi = 5\sin(5\pi/3) = -4.33\ m$

$z = -3\ m$

Point $G(6, 2\pi/3, 8)$

$x = 6\cos(2\pi/3) = -3\ m$

$y = 6\sin(2\pi/3) = 5.196\ m$

$z = 8\ m$

$F_{cart}(2.5, -4.33, -3)\ (m)$ & $G_{cart}(-3, 5.196, 8)\ (m)$

b) the vector component of $\overline{P}$ parallel to $\overline{R}$ with both vectors at point E, and $(2, \pi/3, -3)$

$$\hat{a}_R = \frac{\overline{R}}{|\overline{R}|} = \frac{3\hat{a}_\rho - 6\hat{a}_\phi + 8\hat{a}_z}{\sqrt{3^2 + (-6)^2 + 8^2}} = \frac{3\hat{a}_\rho - 6\hat{a}_\phi + 8\hat{a}_z}{\sqrt{109}}$$

$$\overline{P}_E = -(2)^2\hat{a}_\rho + 3(-3)\hat{a}_\phi - 7\sin\tfrac{\pi}{3}(-3)\hat{a}_z$$

$$= -4\hat{a}_\rho - 9\hat{a}_\phi + 18.1865335\hat{a}_z$$

$$\overline{P}_{aR} = (\overline{P}_E \cdot \hat{a}_R)\hat{a}_R = \left[(-4\hat{a}_\rho - 9\hat{a}_\phi + 18.1865\hat{a}_z)\cdot\hat{a}_R\right]\hat{a}_R$$

$$= \left(\frac{-4(3) - 9(-6) + 18.1865(8)}{\sqrt{109}}\right)\left(\frac{3\hat{a}_\rho - 6\hat{a}_\phi + 8\hat{a}_z}{\sqrt{109}}\right)$$

$$= \frac{187.49227}{109}(3\hat{a}_\rho - 6\hat{a}_\phi + 8\hat{a}_z)$$

$$= 1.72011255(3\hat{a}_\rho - 6\hat{a}_\phi + 8\hat{a}_z)$$

$$\overline{P}_{aR} = 5.1603\,\hat{a}_\rho - 10.3207\,\hat{a}_\phi + 13.7609\,\hat{a}_z$$

c) the distance between points F and G.

$$\overline{r}_F = (2.5\hat{a}_x - 4.33\hat{a}_y - 3\hat{a}_z)\ m$$

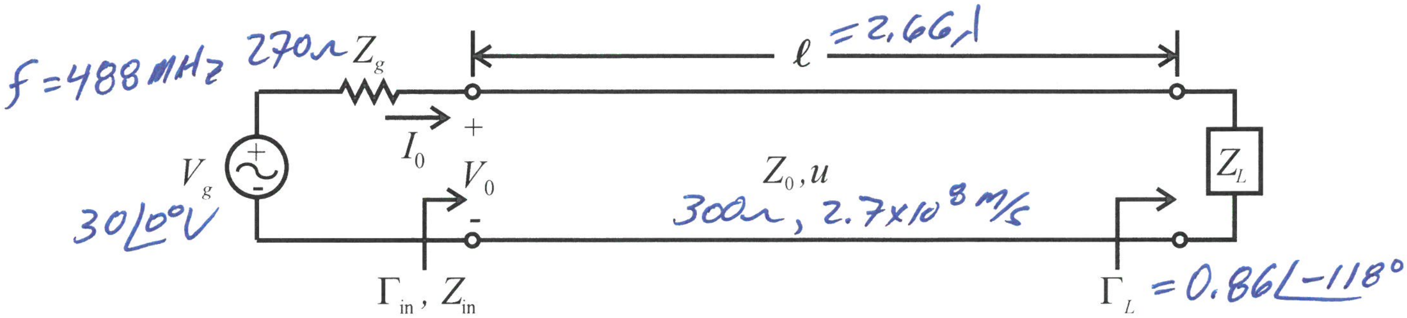
$$\overline{r}_G = (-3\hat{a}_x + 5.196\hat{a}_y + 8\hat{a}_z)\ m$$

$$D_{FG} = |\overline{r}_F - \overline{r}_G| = \sqrt{(2.5+3)^2 + (-4.33-5.196)^2 + (-3-8)^2}$$

$$= \sqrt{241.995} \longrightarrow$$

$$D_{FG} = 15.556\ m$$

4) For the lossless transmission circuit shown, $Z_0 = 300\ \Omega$, $u = 2.7 \times 10^8$ m/s, $\ell = 2.66\lambda$, $f = 488$ MHz, $\Gamma_L = 0.86\angle\text{-}118^\circ$, $Z_g = 270\ \Omega$, and $V_g = 30\ \angle 0^\circ$ (V). Using a **Smith chart (show & label all work)**, find Z_L, Γ_{in}, Z_{in}, VSWR, and Y_{in}. **Express reflection coefficient answers in phasor form A∠θ w/ angle in degrees & admittance/impedance in rectangular format a + *j*b.**



1) Plot $\Gamma_L = 0.86\angle{-118^\circ}$ on Smith chart by drawing a circle of radius 0.86 (use scale @ bottom) and a radial line from center thru -118°. Also, mark SWR (VSWR) scale.

2) Read $\underline{\text{VSWR} = 13.3}$. Read $z_L = 0.1 - j0.6$ Ω/Ω.
$z_L = z_L Z_0 = (0.1 - j0.6)300 = \underline{30 - j180\ \Omega}$.

3) Move $2.66\lambda \to 0.16\lambda$ 'WAVELENGTHS TOWARD GENERATOR' (i.e., $0.16 - 0.086 = 0.074$) on circle of $|\Gamma| = 0.86$ to z_{in}/Γ_{in} point and draw radial line.

4) Read $\underline{\measuredangle \Gamma_{in} = 126.7^\circ}$ and $z_{in} = 0.094 + j0.5$ Ω/Ω.
$Z_{in} = z_{in} Z_0 = (0.094 + j0.5)300 = \underline{28.2 + j150\ \Omega}$.

5) Go 180° around $|\Gamma| = 0.86$ circle from z_{in} to $y_{in} = 0.36 - j1.9$ S/S. $Y_{in} = y_{in}/Z_0$

$$Y_{in} = \frac{0.36 - j1.9}{300} = \underline{0.0012 - j0.0063\ \text{S}}.$$

$Z_L =$ $\underline{30 - j180\ \Omega}$ $\Gamma_{in} =$ $\underline{0.86\angle 126.7^\circ}$ VSWR = $\underline{13.3}$

$Z_{in} =$ $\underline{28.2 + j150\ \Omega}$ $Y_{in} =$ $\underline{0.0012 - j0.0063\ \text{S}}$
$= 1.2 - j6.3$ mS

KEY A

A 5/5

Simple Smith Chart

$Z_0 = 300\ \Omega$
$f = 488$ MHZ
$u = 2.4 \times 10^8$ m/s