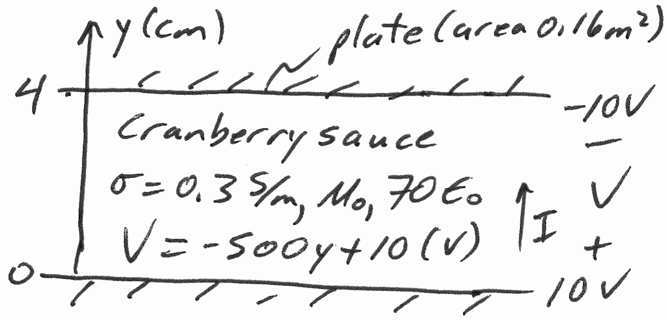
EE 381 Electric & Magnetic Fields Examination #3 (Fall 2021)

Name *Key B*

Instructions: Place answers in <u>indicated</u> spaces. Use notation as given in class for coordinates and vectors. **Show all work** for full credit. Hand-in equation sheet with exam.

1) A delicious lossy capacitor is made with tart cranberry sauce ($\mu = \mu_0$, $\varepsilon = 70\varepsilon_0$, $\sigma = 0.3$ S/m) in the region ($0 < y < 4$ cm) between two large metal plates (area 0.16 m^2). If the electrostatic potential in the sauce is $V = -500\,y + 10$ (V), find the electric field intensity and current density vectors in the sauce as well as the total voltage, current, resistance & capacitance between the plates. Assume fringing is negligible.



$V(0.04\text{m}) = -500(0.04) + 10$
$= -10\text{V}$

$V(0) = -500(0) + 10 = 10\text{V}$

$V_{Tot} = 10 - (-10) = \underline{20\text{V}}$

$$(4.76)\ \bar{E} = -\bar{\nabla} V = -\hat{a}_x \underbrace{\frac{\partial V}{\partial x}}_{\to 0} - \hat{a}_y \frac{\partial(-500y+10)}{\partial y} - \hat{a}_z \underbrace{\frac{\partial V}{\partial z}}_{\to 0}$$

$$= \underline{+\hat{a}_y 500\ \text{V/m}} \text{ in sauce}$$

$$(5.11)\ \bar{J} = \sigma \bar{E} = 0.3(500\hat{a}_y) = \underline{150\,\hat{a}_y\ (\text{A/m}^2)} \text{ in sauce}$$

$$(5.4)\ I = \iint_S \bar{J} \cdot d\bar{s} = \iint_{\substack{\text{plate}\\\text{area}}} 150\hat{a}_y \cdot d\bar{s}_y = 150(0.16) = \underline{24\,\text{A}}$$

(6.16)

$$\text{Ohm's Law}\quad R = V/I = \frac{20}{24} = \underline{0.8\bar{3}\ \Omega}$$

$$(6.35)\ RC = \epsilon/\sigma \Rightarrow C = \frac{\epsilon}{R\sigma} = \frac{70(8.8542\times10^{-12})}{0.8\bar{3}(0.3)}$$

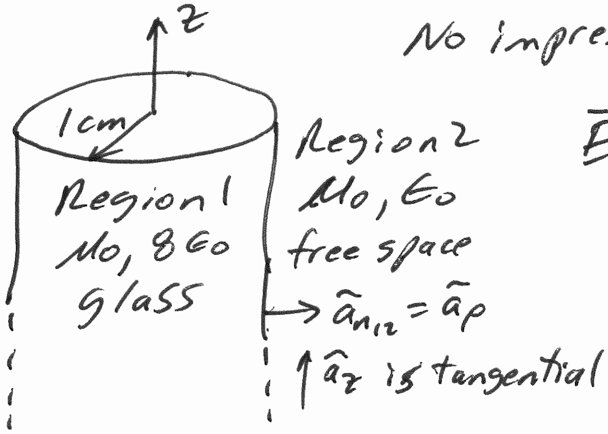
$$= \underline{2.47917\times10^{-9}\,\text{F}}$$

"in sauce" $\equiv$ $0 < y < 4$ cm between $0.16\,\text{m}^2$ plates

$\bar{E}$ = $500\,\hat{a}_y$ (V/m) in sauce $\qquad$ $\bar{J}$ = $150\,\hat{a}_y$ (A/m^2) in sauce

V = 20 V $\qquad$ I = 24 A $\qquad$ R = $0.8\bar{3}\ \Omega$ $\qquad$ C = 2.479 nF

2) Region 1 is a glass stirring rod (μ_0, $8\varepsilon_0$) of radius 1 cm centered on the z-axis surrounded by free space (region 2). If the electric flux density just outside the glass rod is $\bar{D}_2 = 12\,\hat{a}_\rho - 24\,\hat{a}_z$ (nC/m^2), find the electric field & polarization vectors and the electric energy density in both regions near the boundary.



No impressed charges mentioned $\Rightarrow \rho_s = 0$

$$\bar{E}_2 = \frac{\bar{D}_2}{\epsilon_0} = \frac{12\times10^{-9}\hat{a}_\rho - 24\times10^{-9}\hat{a}_z}{8.8542\times10^{-12}}$$

$$= \underline{1355.291\,\hat{a}_\rho - 2710.582\,\hat{a}_z \text{ V/m}}$$

$\underline{\bar{P}_2 = 0}$ (nothing to polarize)

$$(4.97)\ w_{E2} = \tfrac{1}{2}\bar{D}_2\cdot\bar{E}_2 = \tfrac{1}{2}\epsilon_0|\bar{E}_2|^2 = \tfrac{1}{2}(8.8542\times10^{-12})\left[1355.3^2 + (-2710.6)^2\right]$$

$$= \underline{4.06587\times10^{-5}\text{ J/m}^3}$$

Normal boundary condition (5.60) $\bar{D}_{1n} = \bar{D}_{2n} = 12\,\hat{a}_\rho$ (nC/m^2)

$$\bar{E}_{1n} = \bar{D}_{1n}/8\epsilon_0 = \frac{12\times10^{-9}\hat{a}_\rho}{8(8.8542\times10^{-12})} = \underline{169.41136\,\hat{a}_\rho \text{ V/m}}$$

Tangential boundary condition (5.57) $\bar{E}_{1t} = \bar{E}_{2t} = \underline{-2710.6\,\hat{a}_z \text{ V/m}}$

$$\bar{E}_1 = \bar{E}_{1n} + \bar{E}_{1t} = \underline{169.4\,\hat{a}_\rho - 2710.6\,\hat{a}_z \text{ (V/m)}}$$

(5.32) & (5.36) $\bar{P}_1 = \chi_{e1}\epsilon_0\bar{E}_1 = (8-1)\,8.8542\times10^{-12}(169.4\,\hat{a}_\rho - 2710.6\,\hat{a}_z)$

$$= 1.05\times10^{-8}\,\hat{a}_\rho - 1.68\times10^{-7}\,\hat{a}_z \text{ (C/m}^2)$$

$$w_{E1} = \tfrac{1}{2}(8)(8.8542\times10^{-12})\left[169.4114^2 + (-2710.58)^2\right]$$

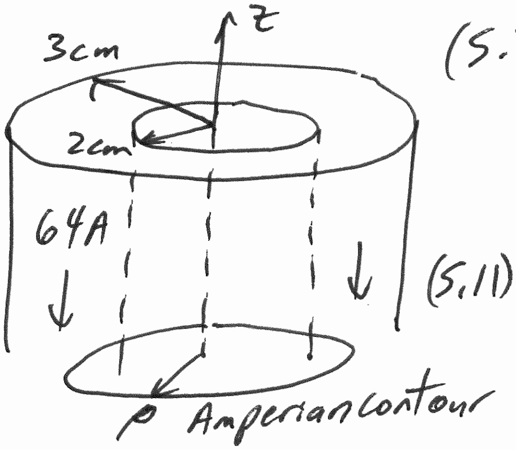
$$= 2.6123\times10^{-4}\text{ J/m}^3$$

$\bar{E}_1 =$ $169.4\,\hat{a}_\rho - 2710.6\,\hat{a}_z$ (V/m) $\quad\bar{P}_1 =$ $10.5\,\hat{a}_\rho - 168\,\hat{a}_z$ (nC/m^2)

$\bar{E}_2 =$ $1355.3\,\hat{a}_\rho - 2710.6\,\hat{a}_z$ (V/m) $\quad\bar{P}_2 =$ 0

$w_{E1} =$ 261.23 (μJ/m^3) $\quad w_{E2} =$ 40.659 (μJ/m^3)

3) In free space, a very long, thick-walled (inner radius of 2 cm and outer radius of 3 cm) copper-iridium alloy ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 3.5 \times 10^7$ S/m) tube is centered on the z-axis and carries a current of 64 A in the $-z$-direction. Given the current is uniformly distributed, find the current density and electric field within the wall of the tube. Then, use Ampere's Law or the Biot-Savart Law to calculate the vector magnetic field at the cylindrical points A(1 cm, 0.4π, 1 cm), B(2.5 cm, 1.2π, -2 cm), and C(4 cm, 0.25π, 0).



(5.2) $J = \dfrac{\Delta I}{\Delta S} = \dfrac{64}{\pi(0.03)^2 - \pi(0.02)^2}$

$= 40{,}743.66544 \ \text{A/m}^2$

$\underline{\bar{J}_{wall} = -40{,}743.665\,\hat{a}_z \ \text{A/m}^2}$

(5.11) $\bar{E}_{wall} = \dfrac{\bar{J}_{wall}}{\sigma} = \dfrac{-40{,}743.665\,\hat{a}_z}{3.5 \times 10^7}$

$\underline{= -1.1641 \times 10^{-3}\,\hat{a}_z \ \text{V/m}}$

Take advantage of cylindrical symmetry to apply Ampere's Law (7.16) + (7.18) $\oint_L \bar{H} \cdot d\bar{\ell} = I_{enc} = \iint_S \bar{J} \cdot d\bar{s}$. Expect $\bar{H} = \hat{a}_\phi H_\phi$. Choose circle of radius ρ as Amperian contour.

$\oint_L \bar{H} \cdot d\bar{\ell} = H_\phi(2\pi\rho) = I_{enc} \Rightarrow \bar{H} = \dfrac{I_{enc}}{2\pi\rho}\,\hat{a}_\phi$

<u>Point A</u> @ $\rho = 1\text{cm}$, $I_{enc} = 0$ $\Rightarrow \bar{H}_A = \dfrac{0}{2\pi(0.01)}\,\hat{a}_\phi = \underline{0}$
no current enclosed

<u>Point B</u> @ $\rho = 2.5\text{cm}$, $I_{enc} = \displaystyle\int_{\phi=0}^{2\pi}\int_{\rho=0.02}^{0.025} -40{,}744\,\hat{a}_z \cdot \hat{a}_z\,\rho\,d\rho\,d\phi$

$= -40{,}744(2\pi - 0)\left(\dfrac{0.025^2}{2} - \dfrac{0.02^2}{2}\right) = \underline{-28.8\,A}$

$\bar{H}_B = \dfrac{-28.8}{2\pi(0.025)}\,\hat{a}_\phi = -183.3465\,\hat{a}_\phi \ (\text{A/m})$

<u>Point C</u> @ $\rho = 4\text{cm}$, $I_{enc} = -64A \Rightarrow \bar{H}_C = \dfrac{-64}{2\pi(0.04)}\,\hat{a}_\phi = \underline{-254.648\,\hat{a}_\phi \ \text{A/m}}$
all enclosed

$\bar{J}_{wall} =$ $-40{,}743.7\,\hat{a}_z \ (\text{A/m}^2)$ $\bar{E}_{wall} =$ $-1.164\,\hat{a}_z \ (\text{mV/m})$ $\bar{H}_A =$ 0

$\bar{H}_B =$ $-183.35\,\hat{a}_\phi \ (\text{A/m})$ $\bar{H}_C =$ $-254.65\,\hat{a}_\phi \ (\text{A/m})$