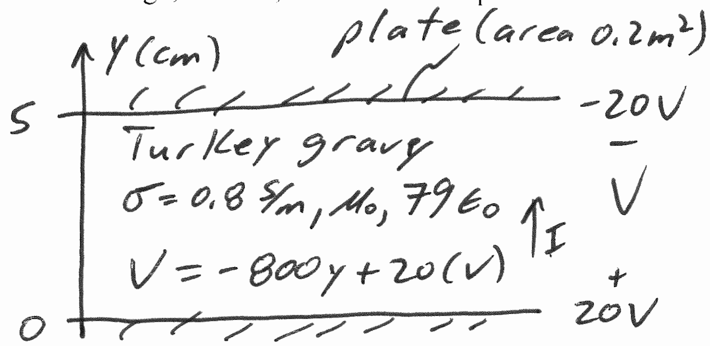
EE 381 Electric & Magnetic Fields Examination #3 (Fall 2021)

Name Key A

Instructions: Place answers in <u>indicated</u> spaces. Use notation as given in class for coordinates and vectors. **Show all work** for full credit. Hand-in equation sheet with exam.

1) A tasty lossy capacitor is made with congealed turkey gravy ($\mu = \mu_0$, $\varepsilon = 79\varepsilon_0$, $\sigma = 0.8$ S/m) in the region ($0 < y < 5$ cm) between two large metal plates (area 0.2 m^2). If the electrostatic potential in the gravy is $V = -800\,y + 20$ (V), find the electric field intensity and current density vectors in the gravy as well as the total voltage, current, resistance & capacitance between the plates. Assume fringing is negligible.



$V(0.05\text{m}) = -800(0.05) + 20 = \underline{-20\text{V}}$

$V(0) = -800(0) + 20 = \underline{20\text{V}}$

$V_{total} = 20 - (-20) = \underline{40\text{V}}$

$$(4.76)\ \bar{E} = -\bar{\nabla} V = -\hat{a}_x \underbrace{\frac{\partial V}{\partial x}}_{0} - \hat{a}_y \frac{\partial(-800y+20)}{\partial y} - \hat{a}_z \underbrace{\frac{\partial V}{\partial z}}_{0}$$

$$= \underline{+\hat{a}_y\, 800\ \text{V/m}} \text{ in gravy}$$

$$(5.11)\ \bar{J} = \sigma \bar{E} = 0.8(800\hat{a}_y) = \underline{640\,\hat{a}_y\ (\text{A/m}^2)} \text{ in gravy}$$

$$(5.4)\ I = \iint_S \bar{J}\cdot d\bar{s} = \iint_{\substack{\text{plate}\\ \text{area}}} 640\hat{a}_y \cdot d\bar{s}_y = 640(0.2) = \underline{128\ \text{A}}$$

(6.16)

$$\text{Ohm's Law } R = V/I = \frac{40}{128} = \underline{0.3125\ \Omega}$$

$$(6.35)\ RC = \epsilon/\sigma \Rightarrow C = \frac{\epsilon}{R\sigma} = \frac{79(8.8542\times 10^{-12})}{0.3125\,(0.8)}$$

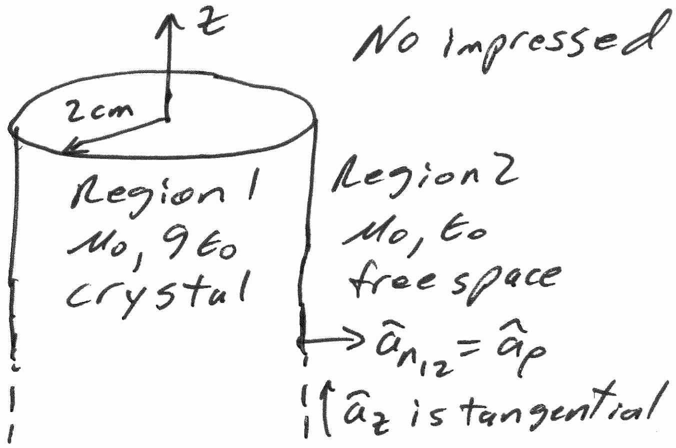
$$= \underline{2.7979\times 10^{-9}\ \text{F}}$$

"in gravy" $\equiv 0 < y < 5$cm between 0.2m² plates

$\bar{E}$ = $800\,\hat{a}_y$ (V/m) in gravy $\bar{J}$ = $640\,\hat{a}_y$ (A/m²) in gravy

V = 40V I = 128 A R = 0.3125 Ω C = 2.798 nF

2) Region 1 is a crystal candlestick (μ_0, $9\varepsilon_0$) of radius 2 cm centered on the z-axis surrounded by free space (region 2). If the electric flux density just outside the glass rod is $\overline{D}_2 = -18\hat{a}_\rho + 28\hat{a}_z$ (nC/m^2), find the electric field & polarization vectors and the electric energy density in both regions near the boundary.



No impressed charges are mentioned $\Rightarrow \rho_s = 0$

$$\overline{E}_2 = \frac{\overline{D}_2}{\epsilon_0} = \frac{-18\times10^{-9}\hat{a}_\rho + 28\times10^{-9}\hat{a}_z}{8.8542\times10^{-12}}$$

$$= -2032.936\,\hat{a}_\rho + 3162.345\,\hat{a}_z \ \frac{V}{m}$$

$\overline{P}_2 = 0$ (nothing to polarize!)

(4.97)

$$W_{E2} = \tfrac{1}{2}\overline{D}_2\cdot\overline{E}_2 = \tfrac{1}{2}\epsilon_0|\overline{E}_2|^2 = \tfrac{1}{2}(8.8542\times10^{-12})\left[(-2032.9)^2 + 3162.3^2\right]$$

$$= 6.2569\times10^{-5}\ \mathrm{J/m^3}$$

Apply normal boundary condition (5.60)

$$\overline{D}_{1n} = \overline{D}_{2n} = -18\,\hat{a}_\rho\ (\mathrm{nC/m^2})$$

$$\hookrightarrow \overline{E}_{1n} = \frac{\overline{D}_{1n}}{9\epsilon_0} = \frac{-18\times10^{-9}\hat{a}_\rho}{9(8.8542\times10^{-12})} = -225.882\,\hat{a}_\rho\ (\mathrm{V/m})$$

Apply tangential boundary condition (5.57)

$$\overline{E}_{1t} = \overline{E}_{2t} = 3162.345\,\hat{a}_z\ (\mathrm{V/m})$$

$$\overline{E}_1 = \overline{E}_{1n} + \overline{E}_{1t} = -225.882\,\hat{a}_\rho + 3162.345\,\hat{a}_z\ \mathrm{V/m}$$

(5.32) & (5.36)

$$\overline{P}_1 = \chi_{e1}\epsilon_0\overline{E} = (9-1)\epsilon_0\overline{E}_1 = 8(8.8542\times10^{-12})(-225.882\,\hat{a}_\rho + 3162.345\,\hat{a}_z)$$

$$= -1.6\times10^{-8}\,\hat{a}_\rho + 2.24\times10^{-7}\,\hat{a}_z\ \mathrm{C/m^2}$$

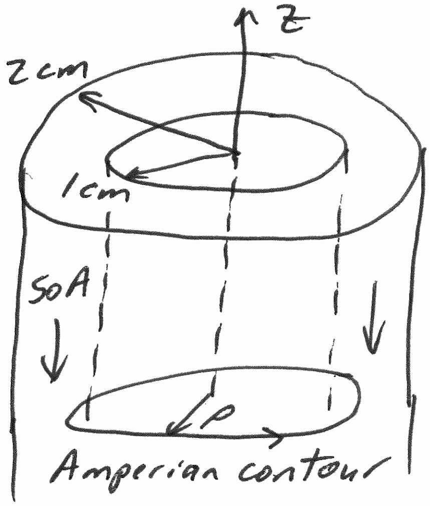
$\overline{E}_1 =$ $-225.9\,\hat{a}_\rho + 3162.3\,\hat{a}_z$ (V/m) $\overline{P}_1 =$ $-16\,\hat{a}_\rho + 224\,\hat{a}_z$ (nC/m^2)

$\overline{E}_2 =$ $-2032.9\,\hat{a}_\rho + 3162.3\,\hat{a}_z$ (V/m) $\overline{P}_2 =$ 0

$w_{E1} =$ 400.49 (μJ/m^3) $w_{E2} =$ 62.569 (μJ/m^3)

$$W_{E1} = \tfrac{1}{2}(9)\,8.8542\times10^{-12}\left[(-225.882)^2 + 3162.345^2\right]$$

3) In free space, a very long, thick-walled (inner radius = 1 cm, outer radius = 2 cm) copper-platinum alloy ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 4.1 \times 10^7$ S/m) pipe is centered on the z-axis and carries a current of 50 A in the $-z$-direction. Given the current is uniformly distributed, find the current density and electric field within the wall of the pipe. Then, use Ampere's Law or the Biot-Savart Law to calculate the vector magnetic field at the cylindrical points A(0.5 cm, 0.5π, 2 cm), B(1.5 cm, 1.5π, -2 cm), and C(3 cm, 0.25π, 0).



(5.2) $J = \frac{\Delta I}{\Delta S} = \frac{50}{\pi(0.02)^2 - \pi(0.01)^2}$

$= 53{,}051.6477 \ A/m^2$

$\underline{\bar{J}_{wall} = -53{,}051.65 \ \hat{a}_z \ A/m^2}$

(5.11) $\bar{E}_{wall} = \frac{\bar{J}_{wall}}{\sigma} = \frac{-53{,}051.65 \ \hat{a}_z}{4.1 \times 10^7}$

$= \underline{-1.29394 \times 10^{-3} \ \hat{a}_z \ V/m}$

Take advantage of cylindrical symmetry to apply Ampere's Law

(7.16) + (7.18) $\oint_L \bar{H} \cdot d\bar{l} = I_{enc} = \iint_S \bar{J} \cdot d\bar{s}$ choose circle of radius ρ for L

$\Rightarrow \oint_L \bar{H} \cdot d\bar{l} = H_\phi (2\pi\rho) = I_{enc} = \iint_S \bar{J} \cdot d\bar{s}$ as we expect $\bar{H} = \hat{a}_\phi H_\phi$

$\hookrightarrow \bar{H} = (I_{enc}/2\pi\rho)\, \hat{a}_\phi$

Point A @ $\rho = 0.5$ cm, $I_{enc} = 0$ $\Rightarrow$ $\underline{\bar{H}_A = \frac{0 \ \hat{a}_\phi}{2\pi(0.005)} = 0}$

No current enclosed

Point B @ $\rho = 1.5$ cm, $I_{enc} = \int_{\phi=0}^{2\pi} \int_{\rho=0.01}^{0.015} -53{,}051.65 \ \hat{a}_z \cdot \hat{a}_z \ \rho \, d\rho \, d\phi$

$= -53{,}051.65 \ (2\pi - 0)\left(\frac{0.015^2}{2} - \frac{0.01^2}{2}\right) = \underline{-20.8\overline{3} \ A}$

$\bar{H}_B = \frac{-20.8\overline{3} \ \hat{a}_\phi}{2\pi(0.015)} = \underline{-221.05 \ \hat{a}_\phi \ (A/m)}$

Point C @ $\rho = 3$ cm, $I_{enc} = -50$ A $\Rightarrow \bar{H}_C = \frac{-50 \ \hat{a}_\phi}{2\pi(0.03)} = \underline{-265.26 \ \hat{a}_\phi \ A/m}$

all enclosed

$\bar{J}_{wall}$ = $-53{,}051.6 \ \hat{a}_z \ (A/m^2)$ $\bar{E}_{wall}$ = $-1.294 \ \hat{a}_z \ (mV/m)$ $\bar{H}_A$ = 0

$\bar{H}_B$ = $-221.05 \ \hat{a}_\phi \ (A/m)$ $\bar{H}_C$ = $-265.26 \ \hat{a}_\phi \ (A/m)$