EE 381 Electric & Magnetic Fields Examination #1 (Fall 2021)

Name Key B

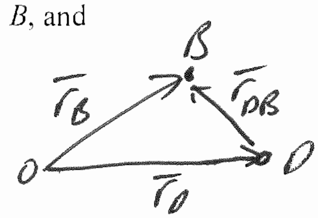
Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheet in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) Given points $A(-2, 7, 8)$, $B(3, 1, -4)$, $C(6, 2, 5)$, & $D(6, -1, 9)$ [units of meters] and vectors $\bar{L} = 60\,\hat{a}_x - 23\,\hat{a}_y + 17\,\hat{a}_z$, $\bar{M} = 12\,\hat{a}_x + 33\,\hat{a}_y - 47\,\hat{a}_z$, and $\bar{N} = 45\,\hat{a}_x + 30\,\hat{a}_y + 50\,\hat{a}_z$, find:

a) the unit vector in the direction pointing from point D to B, and

$$\bar{r}_B = 3\hat{a}_x + \hat{a}_y - 4\hat{a}_z \ (m)$$

$$\bar{r}_D = 6\hat{a}_x - \hat{a}_y + 9\hat{a}_z \ (m)$$



$$\bar{r}_{DB} = \bar{r}_B - \bar{r}_D = (3-6)\hat{a}_x + (1-(-1))\hat{a}_y + (-4-9)\hat{a}_z$$

$$= -3\hat{a}_x + 2\hat{a}_y - 13\hat{a}_z \ (m)$$

$$\hat{a}_{DB} = \frac{\bar{r}_{DB}}{|\bar{r}_{DB}|} = \frac{-3\hat{a}_x + 2\hat{a}_y - 13\hat{a}_z}{\sqrt{(-3)^2 + 2^2 + (-13)^2}} = \frac{-3\hat{a}_x + 2\hat{a}_y - 13\hat{a}_z}{\sqrt{182}}$$

$$\hat{a}_{DB} = -0.2224\hat{a}_x + 0.1482\hat{a}_y - 0.9636\hat{a}_z$$

b) the **scalar** and **vector** components of $\bar{L}$ in the direction of $\bar{N}$. $L_N = \bar{L} \cdot \hat{a}_N$

$$\hat{a}_N = \frac{\bar{N}}{|\bar{N}|} = \frac{45\hat{a}_z + 30\hat{a}_y + 50\hat{a}_z}{\sqrt{45^2 + 30^2 + 50^2}} = 0.61096\hat{a}_x + 0.40731\hat{a}_y + 0.67884\hat{a}_z$$

$$L_N = (60\hat{a}_x - 23\hat{a}_y + 17\hat{a}_z) \cdot (0.611\hat{a}_x + 0.407\hat{a}_y + 0.679\hat{a}_z)$$

$$= \underline{38.82982} = \frac{2860}{73.6546}$$

$$\bar{L}_N = L_N\hat{a}_N = 38.82982(0.61096\hat{a}_x + 0.40731\hat{a}_y + 0.67884\hat{a}_z)$$

$$= 23.72347\hat{a}_x + 15.81577\hat{a}_y + 26.3592\hat{a}_z$$

scalar: 38.83 vector: $23.723\hat{a}_x + 15.816\hat{a}_y + 26.359\hat{a}_z$

2) Given points A_{cart} (-3, 1, 4), B_{cyl} (2, 41°, -3), & C_{sph} (9, 150°, 30°) [units of distance are meters] and vectors $\bar{L} = 8\hat{a}_x$, $\bar{M} = 3\hat{a}_y$, $\bar{N} = -4(x^2 + y^2)\hat{a}_z$, $\bar{P} = 6(x^2 + y^2)\hat{a}_z$, determine:

a) the location of point C in **Cartesian** coordinates $(r = 9m, \theta = 150°, \phi = 30°)$

$$x = r\sin\theta\cos\phi = 9\sin 150°\cos 30° = 3.8971\,m$$

$$y = r\sin\theta\sin\phi = 9\sin 150°\sin 30° = 2.25\,m$$

$$z = r\cos\theta = 9\cos 150° = -7.79423\,m$$

$$C_{cart}(3.897, 2.25, -7.794)\,(m)$$

b) an expression for the vector $\bar{P}$ in **spherical** coordinates, and

$$\bar{P} = 6(x^2+y^2)\hat{a}_z = 6\rho^2\hat{a}_z = 6r^2\sin^2\theta\,\hat{a}_z$$

$$\uparrow$$
$$r = \rho\sin\theta$$

$$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$$

$$\bar{P}_{sph} = 6r^2\sin^2\theta(\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta)$$

$$\bar{P}_{sph} = 6r^2\sin^2\theta(\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta)$$

c) the vector $\bar{M}$ in **cylindrical** coordinates evaluated at point B_{cyl}. $(2m, 41°, -3m)$
$\rho \quad \phi \quad z$

$$\bar{M} = 3\hat{a}_y = 3(\sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi)$$

$$\bar{M}_{Bcyl} = 3(\sin 41°\,\hat{a}_\rho + \cos 41°\,\hat{a}_\phi)$$

$$\bar{M}_{cyl} = 1.968\,\hat{a}_\rho + 2.264\,\hat{a}_\phi$$

$2a = 3mm \quad 2b = 9mm$

3) A Belgian coaxial transmission line is made with brass conductors ($\sigma_b = 1.1\times10^7$ S/m), a center wire of diameter 3 mm and shield of diameter 9 mm, separated by beeswax ($\varepsilon_{bw} = 2.1\varepsilon_0$, $\sigma_{bw} = 2\times10^{-9}$ S/m). It is operated at 210 MHz. Find the skin depth δ as well as the per-unit-length resistance R, inductance L, conductance G, and capacitance C. Next, assuming R & G are negligible, find the phase constant β, phase velocity u, and characteristic impedance Z_0.

Table 11.1 beeswax (insulator): $\mu_{bw} = \mu_0,\ \epsilon_{bw} = 2.1\epsilon_0,\ \sigma_{bw} = 2\times10^{-9}$ S/m

brass (conductor): $\mu_b = \mu_0,\ \epsilon_b = \epsilon_0,\ \sigma_b = 1.1\times10^7$ S/m

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi(210\times10^6)4\pi\times10^{-7}(1.1\times10^7)}} = \underline{10.472\times10^{-6}\ m}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{2\pi\, 1.047\times10^{-6}(1.1\times10^7)}\left(\frac{1}{0.0015}+\frac{1}{0.0045}\right)$$

$$= \underline{1.228176\ \Omega/m}$$

$$L = \frac{\mu_0}{2\pi}\ln(b/a) = \frac{4\pi\times10^{-7}}{2\pi}\ln\left(\frac{4.5}{1.5}\right) = \underline{2.19722\times10^{-7}\ H/m}$$

$$G = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi(2\times10^{-9})}{\ln(4.5/1.5)} = \underline{1.14384\times10^{-8}\ S/m}$$

$$C = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi(2.1)8.8542\times10^{-12}}{\ln(4.5/1.5)} = \underline{1.06342\times10^{-10}\ F/m}$$

$$\beta = 2\pi f\sqrt{LC} = 2\pi(210\times10^6)\sqrt{2.2\times10^{-7}(1.06\times10^{-10})} = \underline{6.37805\ \tfrac{rad}{m}}$$

$$u = 1/\sqrt{LC} = 1/\sqrt{2.2\times10^{-7}(1.06\times10^{-10})} = \underline{2.06876\times10^8\ m/s}$$

$$Z_0 = \sqrt{L/C} = \sqrt{\frac{2.197\times10^{-7}}{1.063\times10^{-10}}} = \underline{45.4554\ \Omega}$$

$\delta =$ 10.472 μm $\quad R =$ 1.2282 Ω/m $\quad L =$ 219.722 nH/m

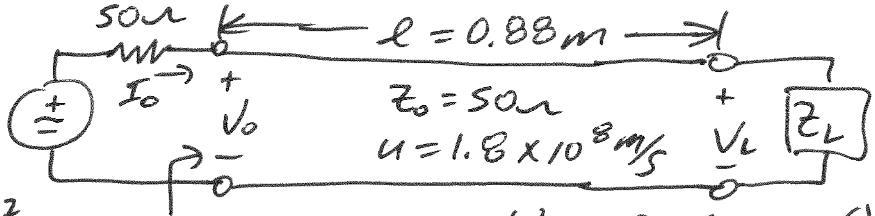
$G =$ 11.438 nS/m $\quad C =$ 106.342 pF/m

$\beta =$ 6.378 rad/m $\quad u =$ 2.0688×10⁸ m/s $\quad Z_0 =$ 45.455 Ω

4) For a lossless transmission circuit, $Z_0 = 50\ \Omega$, $u = 1.8 \times 10^8$ m/s, $\ell = 88$ cm, $f = 320$ MHz, $Z_g = 50\ \Omega$, and $V_g = 30\,\angle 0^\circ$ (V). If an input impedance of $30 + j\,60\ \Omega$ is measured, calculate the phasor input current & voltage, time-average real power delivered at the input & load, and phasor load voltage.

$V_g = 30\angle 0^\circ V$

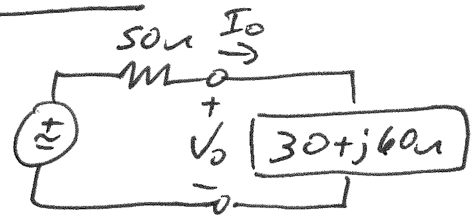
$f = 320\ MHz$



$Z_{in} = 30 + j60\Omega$ $\quad \beta = \frac{\omega}{u} = \frac{2\pi(320\times10^6)}{1.8\times10^8} = 11.1701\ \frac{rad}{m}$

Equiv. Ckt

$30\angle 0^\circ V$



$$I_o = \frac{30\angle 0^\circ}{50 + (30+j60)} = 0.3\angle -36.87^\circ A$$

$$V_o = 0.3\angle -36.87^\circ (30+j60) = 20.125\angle 26.565^\circ\ (V)$$

$$P_{in} = \tfrac{1}{2}Re(V_o I_o^*) = \tfrac{1}{2}Re\{(20.125\angle 26.6^\circ)(0.3\angle +36.87^\circ)\}$$
$$= 1.35W = P_L \quad (\text{Lossless TL!})$$

Need V_o^+ & V_o^-: $V_o^+ = \tfrac{1}{2}(V_o + I_o Z_0) = \tfrac{1}{2}[(20.125\angle 26.6^\circ) + 50(0.3\angle -3$

$$= 15\angle 0^\circ V$$

$$V_o^- = V_o - V_o^+ = 20.125\angle 26.565^\circ - 15\angle 0^\circ = 9.487\angle 71.565^\circ V$$

$$V_L = V_s(z \overset{\ell}{=} 0.88m) = V_o^+ e^{-j\beta\ell} + V_o^- e^{j\beta\ell}$$
$$= 15\angle 0^\circ e^{-j11.17(0.88)} + 9.487\angle 71.565^\circ e^{j11.17(0.88)}$$
$$= 13.4737\angle -164.746^\circ V$$

input current = $0.3\angle -36.87^\circ$ A input voltage = $20.125\angle 26.565^\circ$ V

input power = 1.35 W load power = 1.35 W load voltage = $13.474\angle -164.75^\circ$ V

Express phasor answers in A∠θ form with angle in degrees.