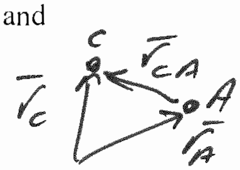
EE 381 Electric & Magnetic Fields Examination #1 (Fall 2021)

Name Key A

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class for coordinates & vectors, & show all work for credit. Insert equation sheet in exam. $\varepsilon_0 = 8.8542\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

1) Given points $A(-2, 7, 8)$, $B(3, 1, -4)$, $C(6, 2, 5)$, and $D(6, -1, 9)$ [units of meters], and vectors $\bar{L} = 60\,\hat{a}_x - 10\,\hat{a}_y + 15\,\hat{a}_z$, $\bar{M} = -12\,\hat{a}_x + 10\,\hat{a}_y - 47\,\hat{a}_z$, and $\bar{N} = 45\,\hat{a}_x + 30\,\hat{a}_y + 52\,\hat{a}_z$ find:

a) the **unit vector** in the direction pointing from point A to point C, and



$$\bar{r}_A = -2\,\hat{a}_x + 7\,\hat{a}_y + 8\,\hat{a}_z \text{ (m)}$$

$$\bar{r}_C = 6\,\hat{a}_x + 2\,\hat{a}_y + 5\,\hat{a}_z \text{ (m)}$$

$$\bar{r}_{AC} = \bar{r}_C - \bar{r}_A = (6-(-2))\,\hat{a}_x + (2-7)\,\hat{a}_y + (5-8)\,\hat{a}_z$$

$$= 8\,\hat{a}_x - 5\,\hat{a}_y - 3\,\hat{a}_z \text{ (m)}$$

$$|\bar{r}_{AC}| = \sqrt{\bar{r}_{AC}\cdot\bar{r}_{AC}} = \sqrt{8^2 + (-5)^2 + (-3)^2} = \sqrt{98}$$

$$\hat{a}_{AC} = \frac{\bar{r}_{AC}}{|\bar{r}_{AC}|} = \frac{8\,\hat{a}_x - 5\,\hat{a}_y - 3\,\hat{a}_z}{\sqrt{98}}$$

$\hat{a}_{AC} = 0.8081\,\hat{a}_x - 0.5051\,\hat{a}_y - 0.303\,\hat{a}_z$

b) the **scalar** and **vector** components of $\bar{M}$ in the direction of $\bar{L}$. $M_L = \bar{M}\cdot\hat{a}_L$

$$\hat{a}_L = \frac{\bar{L}}{|\bar{L}|} = \frac{60\,\hat{a}_x - 10\,\hat{a}_y + 15\,\hat{a}_z}{\sqrt{60^2 + (-10)^2 + 15^2}} = 0.9577\,\hat{a}_x - 0.1596\,\hat{a}_y + 0.2394\,\hat{a}_z$$

$$M_L = \bar{M}\cdot\hat{a}_L = (-12\,\hat{a}_x + 10\,\hat{a}_y - 47\,\hat{a}_z)\cdot(0.9577\,\hat{a}_x - 0.1596\,\hat{a}_y + 0.2394\,\hat{a}_z)$$

$$= -12(0.9577) + 10(-0.1596) - 47(0.2394) = -24.34165 = \frac{-1525}{62.65}$$

$$\bar{M}_L = M_L\,\hat{a}_L = -24.34165\,(0.9577\,\hat{a}_x - 0.1596\,\hat{a}_y + 0.2394\,\hat{a}_z)$$

scalar: -24.342 vector: $-23.312\,\hat{a}_x + 3.885\,\hat{a}_y - 5.828\,\hat{a}_z$

2) Given points $A_{cart}(3, -1, 4)$, $B_{cyl}(2, 71°, -5)$, & $C_{sph}(4, 50°, 250°)$ [units of distance are meters] and vectors $\bar{L} = 3\hat{a}_y$, $\bar{M} = 2\hat{a}_x$, $\bar{N} = -4(x^2+y^2)\hat{a}_z$, $\bar{P} = 6(x^2+y^2)\hat{a}_y$, determine:

a) the location of point C in **Cartesian** coordinates, $(r = 4m,\ \theta = 50°,\ \phi = 250°)$

$$x = r\sin\theta\cos\phi = 4\sin 50°\cos 250° = -1.048\,m$$

$$y = r\sin\theta\sin\phi = 4\sin 50°\sin 250° = -2.8794\,m$$

$$z = r\cos\theta = 4\cos 50° = 2.57115\,m$$

$$C_{cart}(-1.048, -2.879, 2.571)\ (m)$$

b) an expression for the vector $\bar{N}$ in **spherical** coordinates, and

$$\bar{N} = -4(x^2+y^2)\hat{a}_z = -4\rho^2\hat{a}_z = -4r^2\sin^2\theta\,\hat{a}_z$$

$$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$$

$$\bar{N}_{sph} = -4r^2\sin^2\theta(\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta)$$

$$\bar{N}_{sph} = -4r^2\sin^2\theta(\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta)$$

c) the vector $\bar{L}$ in **cylindrical** coordinates evaluated at point $B_{cyl} = (2m, 71°, -5m)$

$$\bar{L} = 3\hat{a}_y = 3(\sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi) = \bar{L}_{cyl}$$

$$\bar{L}_{cyl} = 3(\sin 71°\,\hat{a}_\rho + \cos 71°\,\hat{a}_\phi)$$

$$\bar{L}_{cyl} = 2.8366\,\hat{a}_\rho + 0.9767\,\hat{a}_\phi$$

$2a = 2mm \quad 2b = 10mm$

3) An Albanian coaxial transmission line is made with brass conductors ($\sigma_b = 10^7$ S/m), a center wire of diameter 2 mm and shield of diameter 10 mm, separated by beeswax ($\varepsilon_{bw} = 2.2\varepsilon_0$, $\sigma_{bw} = 10^{-9}$ S/m). It is operated at 220 MHz. Find the skin depth δ as well as the per-unit-length resistance R, inductance L, conductance G, and capacitance C. Next, assuming R & G are negligible, find the phase constant β, phase velocity u, and characteristic impedance Z_0.

beeswax: $\mu_{bw} = \mu_0$, $\epsilon_{bw} = 2.2\epsilon_0$, & $\sigma_{bw} = 10^{-9}$ S/m insulator

brass: $\mu_b = \mu_0$, $\epsilon_b = \epsilon_0$, & $\sigma_b = 10^7$ S/m conductor

Table 11.1

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}} = \frac{1}{\sqrt{\pi (220\times10^6) 4\pi\times10^{-7} (10^7)}} = \underline{10.73022 \times 10^{-6}\ m}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{2\pi\, 1.073\times10^{-5}(10^7)}\left(\frac{1}{10^{-3}} + \frac{1}{5\times10^{-3}}\right)$$

$$= \underline{1.779888\ \Omega/m}$$

$$L = \frac{\mu_0}{2\pi}\ln(b/a) = \frac{4\pi\times10^{-7}}{2\pi}\ln(5/1) = \underline{3.2189\times10^{-7}\ H/m}$$

$$G = \frac{2\pi\sigma_{bw}}{\ln(b/a)} = \frac{2\pi\, 10^{-9}}{\ln(5)} = \underline{3.90396\times10^{-9}\ S/m}$$

$$C = \frac{2\pi\epsilon_{bw}}{\ln(b/a)} = \frac{2\pi\, 2.2(8.8542\times10^{-12})}{\ln(5)} = \underline{76.046\times10^{-12}\ F/m}$$

$$\beta = 2\pi f\sqrt{LC} = 2\pi(220\times10^6)\sqrt{3.2\times10^{-7}(7.6\times10^{-11})} = \underline{6.839\ rad/m}$$

$$u = \frac{1}{\sqrt{LC}} = \left[(3.22\times10^{-7})7.6\times10^{-11}\right]^{-1/2} = \underline{2.0212\times10^8\ m/s}$$

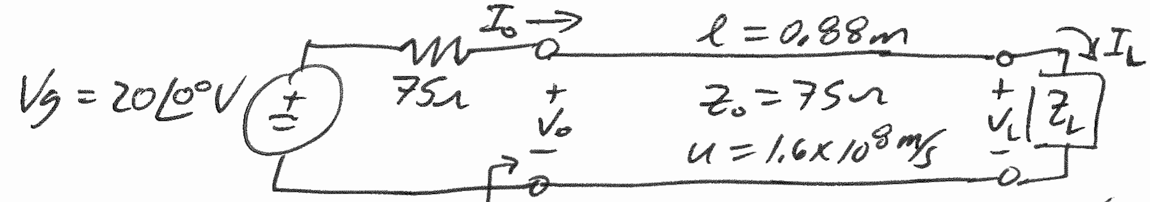
$$Z_0 = \sqrt{L/C} = \sqrt{\frac{3.22\times10^{-7}}{7.6\times10^{-11}}} = \underline{65.05993\ \Omega}$$

$\delta =$ 10.73 μm $\quad R =$ 1.7799 Ω/m $\quad L =$ 321.8876 nH/m

$G =$ 3.904 nS/m $\quad C =$ 76.046 pF/m

$\beta =$ 6.839 rad/m $\quad u =$ 2.0212×10⁸ m/s $\quad Z_0 =$ 65.06 Ω

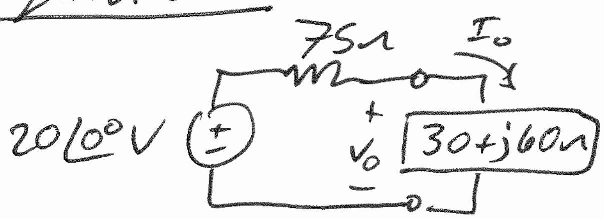
4) For a lossless transmission circuit, $Z_0 = 75\ \Omega$, $u = 1.6 \times 10^8$ m/s, $\ell = 88$ cm, $f = 330$ MHz, $Z_g = 75\ \Omega$, and $V_g = 20\,\angle 0^\circ$ (V). If an input impedance of $30 + j\,60\ \Omega$ is measured, calculate the phasor input current & voltage, time-average real power delivered at the input & load, and phasor load voltage.



$Z_{in} = 30 + j60\Omega$

$\beta = \omega/u = \frac{2\pi(330\times10^6)}{1.6\times10^8} = 12.959$ rad/m

Equiv. ckt



$$I_0 = \frac{20\angle 0^\circ}{75 + (30+j60)} = 0.16538\angle -29.745^\circ A$$

$$V_0 = I_0(30+j60) = 11.094\angle 33.69^\circ V$$

$$P_{in} = \tfrac{1}{2} Re(V_0 I_0^*) = \tfrac{1}{2} Re\{(11.094\angle 33.69^\circ)(0.1654\angle +29.745^\circ)\}$$
$$= 0.4103\ W = P_L \quad (\text{lossless TL!})$$

Need V_0^+ to get V_L: $V_0^+ = \tfrac{1}{2}[V_0 + I_0 Z_0]$
& V_0^-

$$= \tfrac{1}{2}\{11.1\angle 33.7^\circ + 75(0.165\angle -29.745^\circ)\}$$
$$= 10\angle 0^\circ V$$

$$V_0^- = V_0 - V_0^+ = 11.1\angle 33.7^\circ - 10\angle 0^\circ = 6.202\angle 97.125^\circ$$

$$V_L = V_s(z = \ell = 0.88m) = V_0^+ e^{-j\beta\ell} + V_0^- e^{j\beta\ell}$$
$$= 10\angle 0^\circ e^{-j12.959(0.88)} + 6.202\angle 97.125^\circ e^{j12.959(0.88)}$$
$$= 15.4503\angle 52.9281^\circ V$$

input current = $0.1654\angle -29.745^\circ A$ input voltage = $11.094\angle 33.69^\circ V$

input power = $0.4103\ W$ load power = $0.4103\ W$ load voltage = $15.45\angle 52.93^\circ V$

Express phasor answers in A∠θ form with angle in degrees.