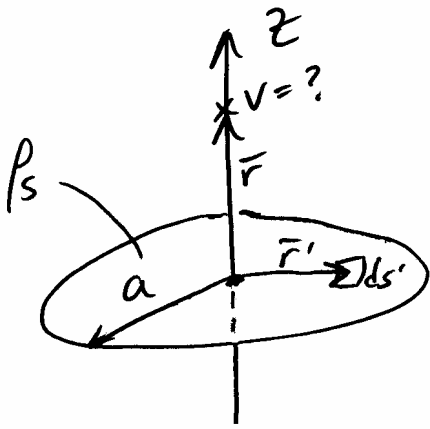


ex. Find potential along  $+z$ -axis due to a uniform surface charge  $\rho_s$  located on a circular disk of radius  $a$  on the  $x$ - $y$  plane.



$$\text{Use } V = \int_S \frac{\rho_s(\vec{r}') ds'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

$$ds' = \rho' d\rho' d\phi' \quad \rho_s(\vec{r}') = \rho_s$$

$$\vec{r} = z \hat{a}_z$$

$$\vec{r}' = \rho' \hat{a}_{\rho'} = \rho' \cos\phi' \hat{a}_x + \rho' \sin\phi' \hat{a}_y$$

$$V = \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{\rho'=0}^a \frac{\rho' d\rho' d\phi'}{|z \hat{a}_z - \rho' \cos\phi' \hat{a}_x - \rho' \sin\phi' \hat{a}_y|}$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{\rho'=0}^a \frac{\rho' d\rho' d\phi'}{\sqrt{z^2 + \rho'^2 \cos^2\phi' + \rho'^2 \sin^2\phi'}}$$

$$\rightarrow \text{Note: } \rho'^2 \cos^2\phi' + \rho'^2 \sin^2\phi' = \rho'^2 [\cos^2\phi' + \sin^2\phi'] \rightarrow 1$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} d\phi' \int_{\rho'=0}^a \frac{\rho' d\rho'}{\sqrt{z^2 + \rho'^2}}$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \phi' \Big|_0^{2\pi} \left( \sqrt{z^2 + \rho'^2} \right) \Big|_{\rho'=0}^a$$

ex. cont.

$$V = \frac{\rho_s}{4\pi\epsilon_0} (2\pi - 0) \left[ \sqrt{z^2 + a^2} - \sqrt{z^2 + 0} \right]$$

$$V = \frac{\rho_s}{2\epsilon_0} \left[ \sqrt{z^2 + a^2} - |z| \right] \text{ (Volts)}$$

OR

$$V = \frac{\rho_s a}{2\epsilon_0} \left[ \sqrt{1 + \left(\frac{z}{a}\right)^2} - \left|\frac{z}{a}\right| \right] \text{ (Volts)}$$

