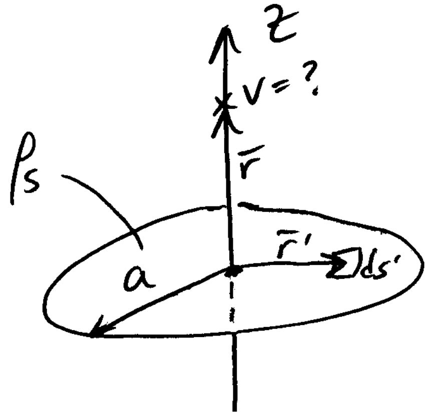
ex. Find potential along +z-axis due to a uniform surface charge ρ_s located on a circular disk of radius a on the x-y plane.



Use $V = \int_S \frac{\rho_s(\bar{r}')\,ds'}{4\pi\epsilon_0 |\bar{r}-\bar{r}'|}$

$ds' = \rho' d\rho' d\phi' \quad \rho_s(\bar{r}') = \rho_s$

$\bar{r} = z\,\hat{a}_z$

$\bar{r}' = \rho'\,\hat{a}_{\rho'} = \rho'\cos\phi'\,\hat{a}_x + \rho'\sin\phi'\,\hat{a}_y$

$$V = \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{\rho'=0}^{a} \frac{\rho' d\rho' d\phi'}{|z\hat{a}_z - \rho'\cos\phi'\hat{a}_x - \rho'\sin\phi'\hat{a}_y|}$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{\rho'=0}^{a} \frac{\rho' d\rho' d\phi'}{\sqrt{z^2 + \rho'^2\cos^2\phi' + \rho'^2\sin^2\phi'}}$$

$\rightarrow$ Note: $\rho'^2\cos^2\phi' + \rho'^2\sin^2\phi' = \rho'^2\underbrace{[\cos^2\phi' + \sin^2\phi']}_{1}$

$$= \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} d\phi' \int_{\rho'=0}^{a} \frac{\rho' d\rho'}{\sqrt{z^2 + \rho'^2}}$$

$$= \frac{\rho_s}{4\pi\epsilon_0}\, \phi'\Big|_0^{2\pi} \left(\sqrt{z^2+\rho'^2}\right)\Big|_{\rho'=0}^{a}$$

ex. cont.

$$V = \frac{\rho_s}{4\pi\epsilon_0}(2\pi - 0)\left[\sqrt{z^2+a^2} - \sqrt{z^2+0}\right]$$

$$V = \frac{\rho_s}{2\epsilon_0}\left[\sqrt{z^2+a^2} - |z|\right] \quad z > 0$$

OR

$$V = \frac{\rho_s a}{2\epsilon_0}\left[\sqrt{1+(z/a)^2} - |z/a|\right] \quad z/a > 0$$