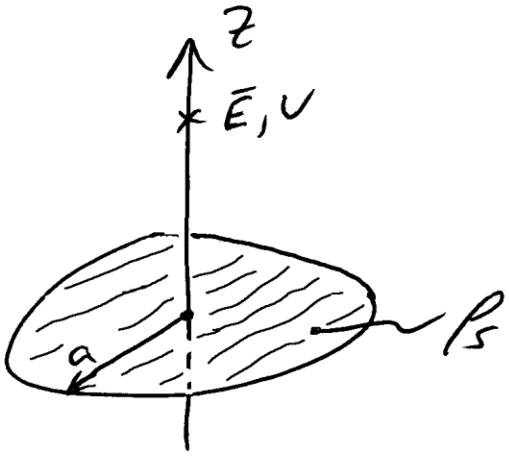
ex. Is the voltage we derived for a circular disk consistent with the electric field we derived earlier?



From prior examples

$$V = \frac{\rho_s}{2\epsilon_0}\left[\sqrt{z^2+a^2} - |z|\right]$$

$$\bar{E} = \hat{a}_z \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]$$

Check

$$\bar{E} = -\bar{\nabla}V = -\left[\hat{a}_\rho \underbrace{\frac{\partial V}{\partial \rho}}_{\to 0} + \hat{a}_\phi \frac{1}{\rho}\underbrace{\frac{\partial V}{\partial \phi}}_{\to 0} + \hat{a}_z \frac{\partial V}{\partial z}\right]$$

$$= -\hat{a}_z \frac{\partial}{\partial z}\left\{\frac{\rho_s}{2\epsilon_0}\left(\sqrt{z^2+a^2} - |z|\right)\right\}$$

$$= -\hat{a}_z \frac{\rho_s}{2\epsilon_0}\left[(1/2)(z^2+a^2)^{-1/2}(2z) - 1\right]$$

$$\bar{E} = \hat{a}_z \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right] \quad \therefore$$

Same Answer!