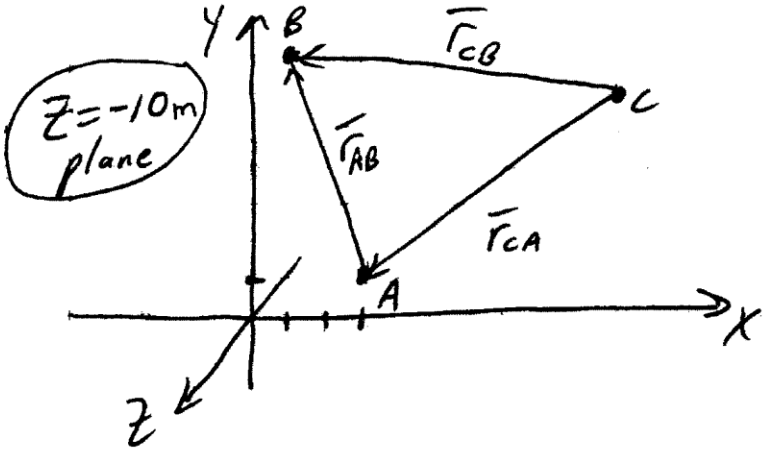
Example- A triangle has vertices located at points *A*(3, 1, -10), *B*(1, 7, -10), and *C*(10, 6, -10) [units of meters]. Find the angle associated with vertex *C*, the area of the triangle, and the perimeter of the triangle.



position vectors

$\bar{r}_A = 3\hat{a}_x + \hat{a}_y - 10\hat{a}_z$ m

$\bar{r}_B = \hat{a}_x + 7\hat{a}_y - 10\hat{a}_z$ m

$\bar{r}_C = 10\hat{a}_x + 6\hat{a}_y - 10\hat{a}_z$ m

distance vectors

→ from C to B

$\bar{r}_{CB} = \bar{r}_B - \bar{r}_C = (1-10)\hat{a}_x + (7-6)\hat{a}_y + (-10-(-10))\hat{a}_z$

$\bar{r}_{CB} = -9\hat{a}_x + \hat{a}_y$ m

→ from C to A

$\bar{r}_{CA} = \bar{r}_A - \bar{r}_C = (3-10)\hat{a}_x + (1-6)\hat{a}_y + (-10+10)\hat{a}_z$

$\bar{r}_{CA} = -7\hat{a}_x - 5\hat{a}_y$ m

→ from A to B

$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (1-3)\hat{a}_x + (7-1)\hat{a}_y + (-10+10)\hat{a}_z$

$\bar{r}_{AB} = -2\hat{a}_x + 6\hat{a}_y$ m

ex. cont.

Using dot product, find θ_c

$$\Rightarrow \bar{r}_{CB} \cdot \bar{r}_{CA} = |\bar{r}_{CB}||\bar{r}_{CA}| \cos\theta_c$$

$$\bar{r}_{CB} \cdot \bar{r}_{CA} = (-9\hat{a}_x + \hat{a}_y) \cdot (-7\hat{a}_x - 5\hat{a}_y) = -9 \cdot (-7) + 1(-5) = \underline{58}$$

$$|\bar{r}_{CB}| = \sqrt{\bar{r}_{CB} \cdot \bar{r}_{CB}} = \sqrt{(-9)^2 + 1^2} = \underline{\sqrt{82}}$$

$$|\bar{r}_{CA}| = \sqrt{\bar{r}_{CA} \cdot \bar{r}_{CA}} = \sqrt{(-7)^2 + (-5)^2} = \underline{\sqrt{74}}$$

$$\cos\theta_c = \frac{\bar{r}_{CB} \cdot \bar{r}_{CA}}{|\bar{r}_{CB}||\bar{r}_{CA}|} = \frac{58}{\sqrt{82}\sqrt{74}} = 0.74457$$

$$\underline{\theta_c = \cos^{-1}(0.74457) = 41.878^\circ}$$

$$\bar{r}_{CA} \times \bar{r}_{AB} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -7 & -5 & 0 \\ -2 & 6 & 0 \end{vmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ -7 & -5 \\ -2 & 6 \end{matrix}$$

$$= (\hat{a}_x 0 + \hat{a}_y 0 - \hat{a}_z 42) - (\hat{a}_x 0 + \hat{a}_y 0 + \hat{a}_z 10)$$

$$= \hat{a}_z(-52)$$

triangle area $= \frac{1}{2}$ (base)(height)

$$= \frac{1}{2} \left| \text{side dist vector}_i \times \text{side dist vector}_j \right|$$

$$= \frac{1}{2} \left| \bar{r}_{CA} \times \bar{r}_{AB} \right| = \frac{52}{2} = \underline{26\ m^2}$$

ex. cont.

$$\text{perimeter} = |\bar{r}_{AB}| + |\bar{r}_{CB}| + |\bar{r}_{CA}|$$

$$= \sqrt{(-2)^2+6^2} + \sqrt{(-9)^2+1^2} + \sqrt{(-7)^2+(-5)^2}$$

$$= \sqrt{40} + \sqrt{82} + \sqrt{74}$$

$$\text{perimeter} = 23.982 \text{ m}$$