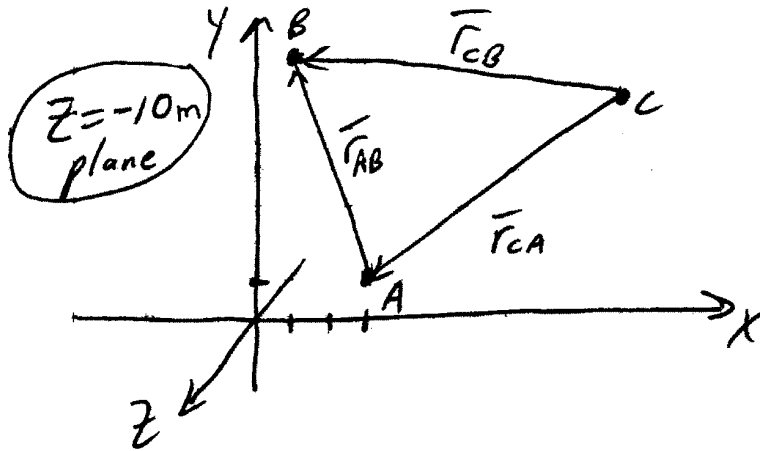**Example-** A triangle has vertices located at points $A(3, 1, -10)$, $B(1, 7, -10)$, and $C(10, 6, -10)$ [units of meters]. Find the angle associated with vertex C, the area of the triangle, and the perimeter of the triangle.

position vectors

$$\bar{r}_A = 3\hat{a}_x + \hat{a}_y - 10\hat{a}_z \quad m$$

$$\bar{r}_B = \hat{a}_x + 7\hat{a}_y - 10\hat{a}_z \quad m$$

$$\bar{r}_C = 10\hat{a}_x + 6\hat{a}_y - 10\hat{a}_z \quad m$$

distance vectors

→ from C to B

$$\bar{r}_{CB} = \bar{r}_B - \bar{r}_C = (1-10)\hat{a}_x + (7-6)\hat{a}_y + (-10-(-10))\hat{a}_z$$

$$\bar{r}_{CB} = -9\hat{a}_x + \hat{a}_y \quad m$$

→ from C to A

$$\bar{r}_{CA} = \bar{r}_A - \bar{r}_C = (3-10)\hat{a}_x + (1-6)\hat{a}_y + (-10+10)\hat{a}_z$$

$$\bar{r}_{CA} = -7\hat{a}_x - 5\hat{a}_y \quad m$$

→ from A to B

$$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (1-3)\hat{a}_x + (7-1)\hat{a}_y + (-10+10)\hat{a}_z$$

$$\bar{r}_{AB} = -2\hat{a}_x + 6\hat{a}_y \quad m$$

ex. cont.

Using dot product, find θ_c

$$\Rightarrow \bar{r}_{CB} \cdot \bar{r}_{CA} = |\bar{r}_{CB}||\bar{r}_{CA}| \cos \theta_c$$

$$\bar{r}_{CB} \cdot \bar{r}_{CA} = (-9\hat{a}_x + \hat{a}_y) \cdot (-7\hat{a}_x - 5\hat{a}_y) = -9(-7) + 1(-5) = \underline{58}$$

$$|\bar{r}_{CB}| = \sqrt{\bar{r}_{CB} \cdot \bar{r}_{CB}} = \sqrt{(-9)^2 + 1^2} = \underline{\sqrt{82}}$$

$$|\bar{r}_{CA}| = \sqrt{\bar{r}_{CA} \cdot \bar{r}_{CA}} = \sqrt{(-7)^2 + (-5)^2} = \underline{\sqrt{74}}$$

$$\cos \theta_c = \frac{\bar{r}_{CB} \cdot \bar{r}_{CA}}{|\bar{r}_{CB}||\bar{r}_{CA}|} = \frac{58}{\sqrt{82}\sqrt{74}} = 0.74457$$

$$\underline{\underline{\theta_c = \cos^{-1}(0.74457) = 41.878°}}$$

$$\bar{r}_{CA} \times \bar{r}_{AB} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -7 & -5 & 0 \\ -2 & 6 & 0 \end{vmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ -7 & -5 \\ -2 & 6 \end{matrix}$$

$$= (\hat{a}_x 0 + \hat{a}_y 0 - \hat{a}_z 42) - (\hat{a}_x 0 + \hat{a}_y 0 + \hat{a}_z 10)$$

$$= \hat{a}_z (-52) \, m^2$$

triangle area $= \frac{1}{2} (base)(height)$

$$= \frac{1}{2} | \text{side dist. vector}_i \times \text{side dist. vector}_j |$$

$$= \frac{1}{2} | \bar{r}_{CA} \times \bar{r}_{AB} | = \frac{52}{2} = \underline{\underline{26 \, m^2}}$$

ex. cont.

$$\text{perimeter} = |\overline{r}_{AB}| + |\overline{r}_{CB}| + |\overline{r}_{CA}|$$

$$= \sqrt{(-2)^2 + 6^2} + \sqrt{(-9)^2 + 1^2} + \sqrt{(-7)^2 + (-5)^2}$$

$$= \sqrt{40} + \sqrt{82} + \sqrt{74}$$

$$\underline{\text{perimeter} = 23.982 \ m}$$