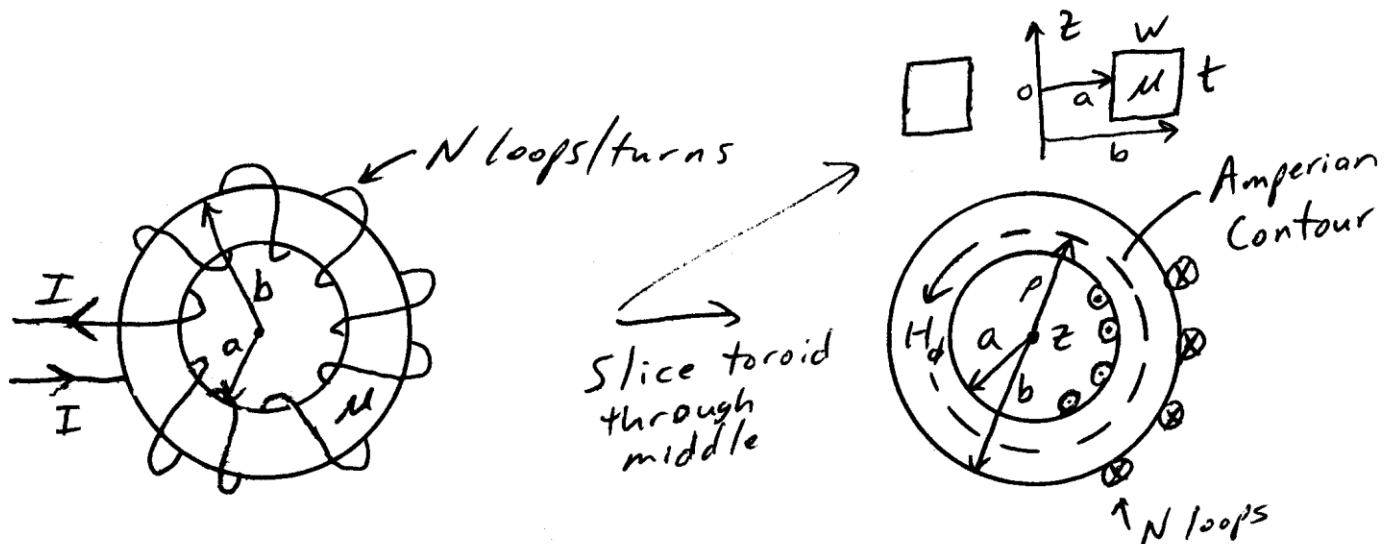


Example- For a toroidal inductor with N turns of wire tightly wound around a core (μ) with a rectangular cross-section ($w \times t$), find the magnetic field and magnetic flux density within the toroid. Then, find the inductance.



By symmetry, we expect $\vec{H} = \hat{a}_\phi H_\phi$ and choose an Amperian contour to be a circle of radius ρ where $a < \rho < b$ and at height $-\frac{t}{2} < z < \frac{t}{2}$.

$$\text{Then, } \oint_C \vec{H} \cdot d\vec{\ell} = \int H_\phi d\ell = H_\phi (2\pi\rho)$$

$$I_{\text{enc}} = NI \quad \leftarrow \text{By RHR } I \text{ is positive}$$

$$\hookrightarrow H_\phi (2\pi\rho) = NI \quad \Rightarrow \quad H_\phi = \frac{NI}{2\pi\rho}$$

$$\vec{H} = \hat{a}_\phi \frac{NI}{2\pi\rho} \quad \left. \begin{array}{l} a < \rho < b \\ -\frac{t}{2} < z < \frac{t}{2} \end{array} \right\} \begin{array}{l} \text{inside} \\ \text{toroid} \end{array}$$

$$\vec{B} = \mu \vec{H} = \hat{a}_\phi \frac{\mu NI}{2\pi\rho} \quad \begin{array}{l} a < \rho < b \\ -\frac{t}{2} < z < \frac{t}{2} \end{array}$$

use \bar{B} to find Ψ_m for toroid through a single loop

$$\Psi_m = \iiint_{\substack{\text{loop} \\ \text{cross} \\ \text{section}}} \bar{B} \cdot d\bar{s} = \int_{z=-t/2}^{t/2} \int_{\rho=a}^b \hat{a}_\phi \frac{\mu N I}{2\pi \rho} \cdot \hat{a}_\phi \rho d\rho dz$$

$$= \frac{\mu N I}{2\pi} \int_{-t/2}^{t/2} dz \int_a^b \frac{d\rho}{\rho} = \frac{\mu N I}{2\pi} (z) \Big|_{-t/2}^{t/2} (\ln \rho) \Big|_a^b$$

$$\Psi_m = \frac{\mu N I t}{2\pi} \ln(b/a)$$

$$\lambda = N \Psi_m = \frac{\mu N^2 I t}{2\pi} \ln(b/a)$$

$$L = \frac{\lambda}{I} = \frac{\mu N^2 t}{2\pi} \ln(b/a)$$

If $w \ll a$, use $b = a + w$ and $\ln(1+x) = x - \frac{x^2}{2} + \dots$

$$L \approx \frac{\mu N^2 \overbrace{tw}^{\text{cross-sectional Area}}}{2\pi a} = \frac{\mu N^2 A}{2\pi a}$$

useful for toroids w/ non-rectang. cross-sections