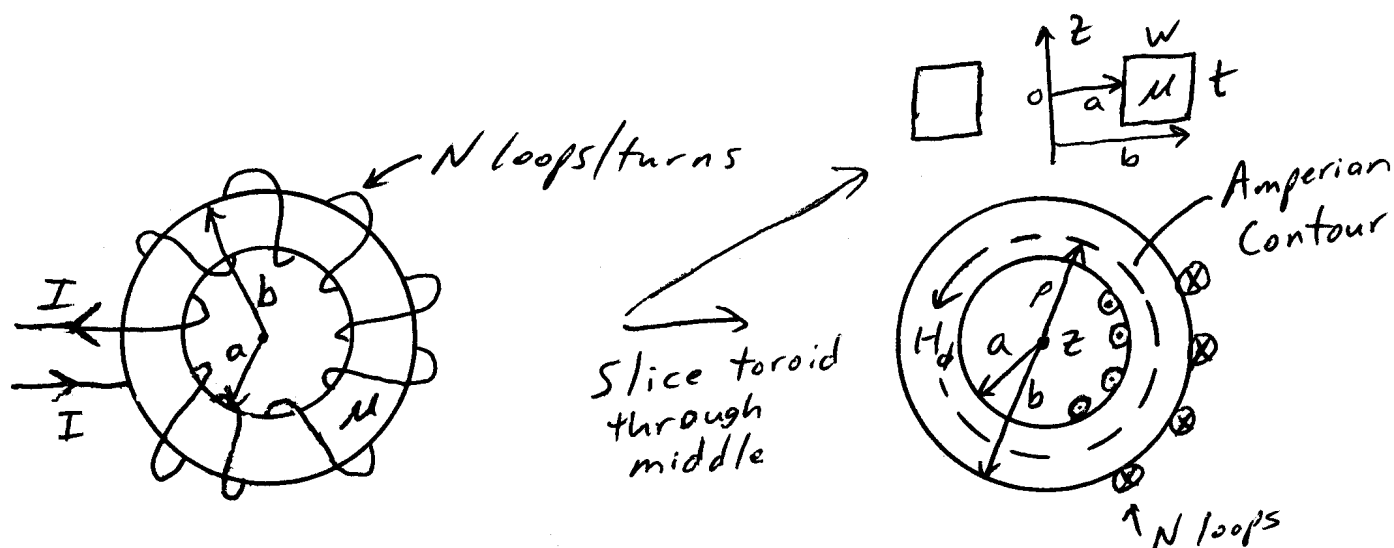


**Example-** For a toroidal inductor with  $N$  turns of wire tightly wound around a core ( $\mu$ ) with a rectangular cross-section ( $w \times t$ ), find the magnetic field and magnetic flux density within the toroid. Then, find the inductance.



By symmetry, we expect  $\vec{H} = \hat{a}_\phi H_\phi$  and choose an Amperian contour to be a circle of radius  $\rho$  where  $a < \rho < b$  and at height  $-\frac{t}{2} < z < \frac{t}{2}$ .

$$\text{Then, } \oint_C \vec{H} \cdot d\vec{\ell} = \int H_\phi d\ell = H_\phi (2\pi\rho)$$

$$I_{\text{enc}} = NI \quad \leftarrow \text{By RHR } I \text{ is positive}$$

$$\hookrightarrow H_\phi (2\pi\rho) = NI \Rightarrow H_\phi = \frac{NI}{2\pi\rho}$$

$$\vec{H} = \hat{a}_\phi \frac{NI}{2\pi\rho} \quad \left. \begin{array}{l} a < \rho < b \\ -\frac{t}{2} < z < \frac{t}{2} \end{array} \right\} \begin{array}{l} \text{inside} \\ \text{toroid} \end{array}$$

$$\vec{B} = \mu \vec{H} = \hat{a}_\phi \frac{\mu NI}{2\pi\rho} \quad \begin{array}{l} a < \rho < b \\ -\frac{t}{2} < z < \frac{t}{2} \end{array}$$

Use  $\vec{B}$  to find  $\Psi_m$  for toroid through a single loop

$$\Psi_m = \iint_{\substack{\text{loop} \\ \text{cross} \\ \text{section}}} \vec{B} \cdot d\vec{s} = \int_{z=-t/2}^{t/2} \int_{r=a}^b \hat{a}_\phi \frac{\mu N I}{2\pi r} \cdot \hat{a}_\phi r dr dz$$

$$= \frac{\mu N I}{2\pi} \int_{-t/2}^{t/2} dz \int_a^b \frac{dr}{r} = \frac{\mu N I}{2\pi} (z) \Big|_{-t/2}^{t/2} (\ln r) \Big|_a^b$$

$$\underline{\underline{\Psi_m = \frac{\mu N I t}{2\pi} \ln(b/a)}}$$

$$\lambda = N \Psi_m = \frac{\mu N^2 I t}{2\pi} \ln(b/a)$$

$$\underline{\underline{L = \frac{\lambda}{I} = \frac{\mu N^2 t}{2\pi} \ln(b/a)}}$$

If  $w \ll a$ , use  $b = a + w$  and  $\ln(1+x) = x - \frac{x^2}{2} + \dots$

$$\Rightarrow \ln\left(\frac{b}{a}\right) = \ln\left(1 + \frac{w}{a}\right) \approx \frac{w}{a}$$

$$\underline{\underline{L \approx \frac{\mu N^2 \overbrace{tw}^{\text{cross-sectional Area}}}{2\pi a} = \frac{\mu N^2 A}{2\pi a}}}$$

useful for toroids w/ non-rectang. cross-sections