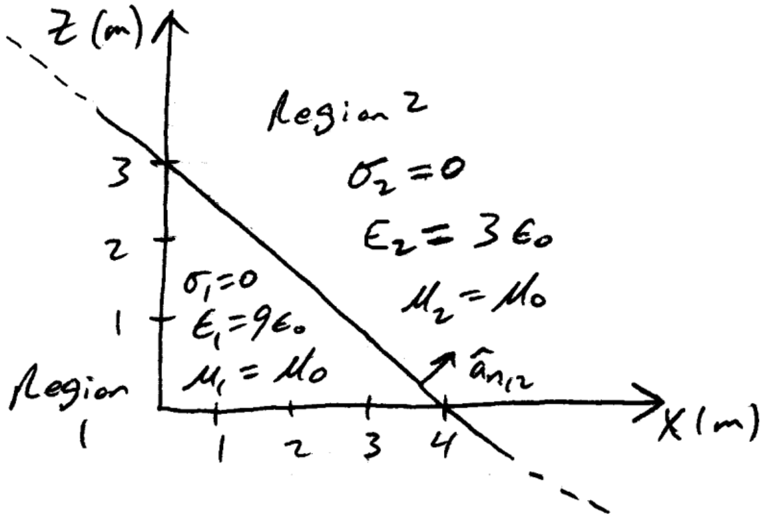
ex. Given $\bar{E}_1 = 100\hat{a}_x + 300\hat{a}_y$ V/m, find the current densities in both regions and the electric field in region 2 and the charge density at the boundary.



Since $\sigma_1 = \sigma_2 = 0$

$$\underline{\underline{\bar{J}_1 = \bar{J}_2 = \sigma\bar{E} = 0}}$$

Since no free charges mentioned + $\sigma_1 = \sigma_2 = 0$

$$\Rightarrow \underline{\underline{\rho_s = 0}}$$

To find $\hat{a}_{n12}$, either use inspection/geometry or the gradient function.

plane/line eqn $z = -\frac{3}{4}x + 3 \rightarrow 4z = -3x + 12$

$$f = 4z + 3x = 12$$

$$\bar{N}_{12} = \pm\bar{\nabla}f = \pm\left\{\frac{\partial(f)}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z\right\}$$

$$= \pm\{3\hat{a}_x + 4\hat{a}_z\}$$ $\leftarrow$ use positive direction

$$\hat{a}_{n12} = \frac{3\hat{a}_x + 4\hat{a}_z}{\sqrt{3^2 + 4^2}} = \underline{0.6\hat{a}_x + 0.8\hat{a}_z}$$

$$\bar{E}_{1n} = \hat{a}_{n12}(\bar{E}_1 \cdot \hat{a}_{n12}) = (0.6\hat{a}_x + 0.8\hat{a}_z)(60)$$

$$\underline{\bar{E}_{1n} = 36\hat{a}_x + 48\hat{a}_z} \text{ V/m}$$

ex. cont.

$$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n} = (100\hat{a}_x + 300\hat{a}_y) - (36\hat{a}_x + 48\hat{a}_z)$$

$$\underline{\bar{E}_{1t} = 64\,\hat{a}_x + 300\,\hat{a}_y - 48\,\hat{a}_z} \quad V/m$$

Apply Tangential Boundary Condition

$$\bar{E}_{2t} = \bar{E}_{1t} = 64\,\hat{a}_x + 300\,\hat{a}_y - 48\,\hat{a}_z \ V/m$$

Apply Normal Boundary Condition

$$\bar{D}_{2n} = \bar{D}_{1n} \quad (\text{since } \rho_s = 0)$$

$$3\epsilon_0 \bar{E}_{2n} = 9\epsilon_0 \bar{E}_{1n}$$

$$\bar{E}_{2n} = \frac{9}{3}\,\bar{E}_{1n} = 3\left[36\,\hat{a}_x + 48\,\hat{a}_z\right]$$

$$\underline{\bar{E}_{2n} = 108\,\hat{a}_x + 144\,\hat{a}_z \ V/m}$$

$$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

$$= (64\hat{a}_x + 300\hat{a}_y - 48\hat{a}_z) + (108\,\hat{a}_x + 144\hat{a}_z)$$

$$\underline{\underline{\bar{E}_2 = 172\,\hat{a}_x + 300\,\hat{a}_y + 96\,\hat{a}_z \quad V/m}}$$