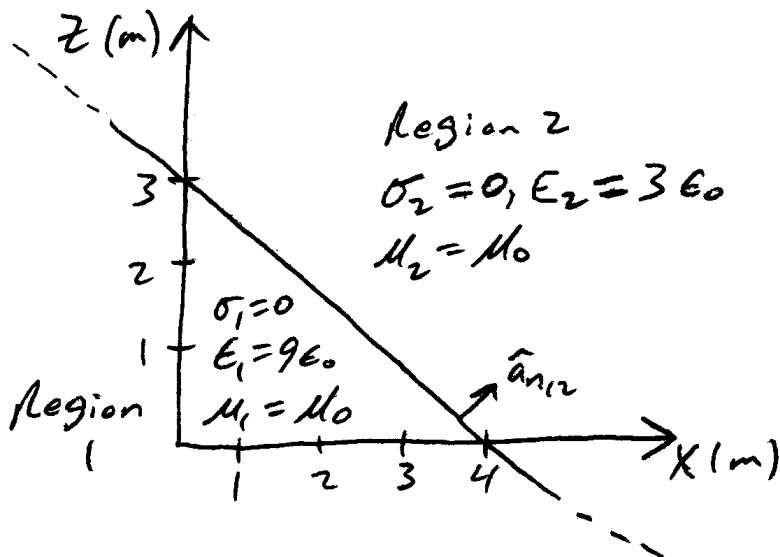


**Example-** Near the dielectric-dielectric interface shown,  $\vec{E}_1 = 100\hat{a}_x + 300\hat{a}_y$  (V/m).

Find the vector current densities in both regions and the vector electric field in region 2 and the charge density at the interface.



Since  $\sigma_1 = \sigma_2 = 0$ ,

$$\vec{J}_1 = \vec{J}_2 = \sigma \vec{E} = 0$$

Since no free charges mentioned +  $\sigma_1 = \sigma_2 = 0$

$$\Rightarrow \underline{\underline{\rho_s = 0}}$$

To find  $\hat{a}_{n12}$ , either use inspection/geometry or the gradient function.

plane/line eqn  $z = -\frac{3}{4}x + 3 \rightarrow 4z = -3x + 12$

$$f = 4z + 3x - 12 = 0$$

$$\begin{aligned} \vec{N}_{12} &= \pm \vec{\nabla} f = \pm \left\{ \frac{\partial f}{\partial x} \hat{a}_x + \frac{\partial f}{\partial y} \hat{a}_y + \frac{\partial f}{\partial z} \hat{a}_z \right\} \\ &= \pm \{ 3\hat{a}_x + 4\hat{a}_z \} \quad \leftarrow \text{use positive direction} \end{aligned}$$

$$\hat{a}_{n12} = \frac{3\hat{a}_x + 4\hat{a}_z}{\sqrt{3^2 + 4^2}} = \underline{\underline{0.6\hat{a}_x + 0.8\hat{a}_z}}$$

$$\vec{E}_{1n} = \hat{a}_{n12} (\vec{E}_1 \cdot \hat{a}_{n12}) = (0.6\hat{a}_x + 0.8\hat{a}_z) (0.6(100))$$

$$\underline{\underline{\vec{E}_{1n} = 36\hat{a}_x + 48\hat{a}_z \text{ V/m}}}$$

ex. cont.

$$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n} = (100\hat{a}_x + 300\hat{a}_y) - (36\hat{a}_x + 48\hat{a}_z)$$

$$\bar{E}_{1t} = 64\hat{a}_x + 300\hat{a}_y - 48\hat{a}_z \text{ V/m}$$

Apply Tangential Boundary Condition

$$\bar{E}_{2t} = \bar{E}_{1t} = 64\hat{a}_x + 300\hat{a}_y - 48\hat{a}_z \text{ V/m}$$

Apply Normal Boundary Condition

$$\bar{D}_{2n} = \bar{D}_{1n} \quad (\text{since } \rho_s = 0)$$

$$3\epsilon_0 \bar{E}_{2n} = 9\epsilon_0 \bar{E}_{1n}$$

$$\bar{E}_{2n} = \frac{9}{3} \bar{E}_{1n} = 3[36\hat{a}_x + 48\hat{a}_z]$$

$$\bar{E}_{2n} = 108\hat{a}_x + 144\hat{a}_z \text{ V/m}$$

$$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

$$= (64\hat{a}_x + 300\hat{a}_y - 48\hat{a}_z) + (108\hat{a}_x + 144\hat{a}_z)$$

$$\bar{E}_2 = 172\hat{a}_x + 300\hat{a}_y + 96\hat{a}_z \text{ V/m}$$