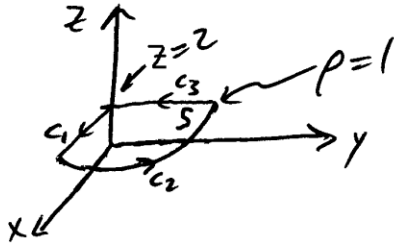


Verify Stoke's Theorem, $\int_S (\nabla \times \bar{A}) \cdot d\bar{S} = \oint_c \bar{A} \cdot d\bar{l}$, for the vector field $\bar{F} = \hat{a}_\rho + z\hat{a}_\phi + \hat{a}_z$ for the contour c surrounding surface S described by $0 \leq \rho \leq 1$, $0 \leq \phi \leq \pi/2$, and $z = 2$.



LHS

$$\begin{aligned} \nabla \times \bar{F} &= \hat{a}_\rho \left(\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) + \hat{a}_\phi \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \right) + \hat{a}_z \left(\frac{1}{\rho} \frac{\partial(\rho F_\phi)}{\partial \rho} - \frac{1}{\rho} \frac{\partial F_\rho}{\partial \phi} \right) \\ &= \underline{-\hat{a}_\rho + \hat{a}_z \frac{1}{\rho} z} \end{aligned}$$

$$\begin{aligned} \int_S (\nabla \times \bar{F}) \cdot d\bar{S}_2 &= \int_{\rho=0}^1 \int_{\phi=0}^{\pi/2} (-\hat{a}_\rho + \hat{a}_z \frac{z}{\rho}) \cdot \hat{a}_z \rho d\phi d\rho \\ &= \int_{\rho=0}^1 \int_{\phi=0}^{\pi/2} z d\phi d\rho = z(1-0)(\pi/2 - 0) \\ &= \underline{\underline{\pi = 3.1416}} \end{aligned}$$

RHS

$$\begin{aligned} \oint_c \bar{F} \cdot d\bar{l} &= \int_{c_1} (\hat{a}_\rho + z\hat{a}_\phi + \hat{a}_z) \cdot (d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z) \\ &\quad + \int_{c_2} (\hat{a}_\rho + z\hat{a}_\phi + \hat{a}_z) \cdot (d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z) \\ &\quad + \int_{c_3} (\hat{a}_\rho + z\hat{a}_\phi + \hat{a}_z) \cdot (d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z) \\ &= \int_{\rho=0}^1 d\rho + \int_{\phi=0}^{\pi/2} z d\phi + \int_{\rho=0}^1 d\rho = (\pi/2 - 0)z \\ \oint_c \bar{F} \cdot d\bar{l} &= \underline{\underline{\pi = 3.1416}} \end{aligned}$$