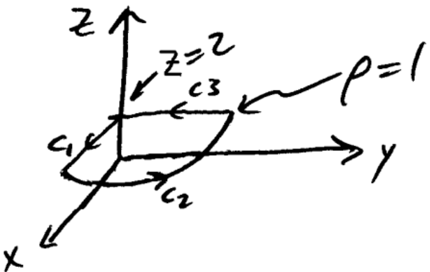
Stoke's Theorem- $\oint_c \bar{A}\bullet d\bar{l} = \int_s (\bar{\nabla}\times\bar{A})\bullet d\bar{s}$

ex. Verify Stoke's Thm

for $\bar{F} = \hat{a}_\rho + z\,\hat{a}_\phi + \hat{a}_z$



LHS

$$\bar{\nabla}\times\bar{F} = \hat{a}_\rho\left(\frac{1}{\rho}\overset{\nearrow 0}{\cancel{\frac{\partial F_z}{\partial\phi}}} - \underset{\searrow 1}{\cancel{\frac{\partial F_\phi}{\partial z}}}\right) + \hat{a}_\phi\left(\underset{\searrow 0}{\cancel{\frac{\partial F_\rho}{\partial z}}} - \underset{\searrow 0}{\cancel{\frac{\partial F_z}{\partial \rho}}}\right)$$

$$+\hat{a}_z\left(\frac{1}{\rho}\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{1}{\rho}\underset{\searrow 0}{\cancel{\frac{\partial F_\rho}{\partial\phi}}}\right)$$

$$= \underline{-\hat{a}_\rho + \hat{a}_z \frac{1}{\rho} z}$$

$$\int_s (\bar{\nabla}\times\bar{F})\cdot d\bar{s}_z = \int_{\rho=0}^{1}\int_{\phi=0}^{\pi/2}\left(-\hat{a}_\rho + \hat{a}_z\frac{z}{\rho}\right)\cdot\hat{a}_z\,\rho\, d\phi\, d\rho$$

$$= \int_{\rho=0}^{1}\int_{\phi=0}^{\pi/2}\overset{\nearrow 2}{\cancel{z}}\,d\phi\, d\rho = 2(1-0)(\pi/2 - 0)$$

$$= \underline{\underline{\pi}}$$

RHS

$$\oint_c \bar{F}\cdot d\bar{l} = \int_{c_1}(\hat{a}_\rho + \overset{2}{\cancel{z}}\hat{a}_\phi + \hat{a}_z)\cdot(d\rho\,\hat{a}_\rho + \rho\, \overset{0}{\cancel{d\phi}}\hat{a}_\phi + \overset{0}{\cancel{dz}}\hat{a}_z)$$

$$+\int_{c_2}(\hat{a}_\rho + \overset{2}{\cancel{z}}\hat{a}_\phi + \hat{a}_z)\cdot(\overset{0}{\cancel{d\rho}}\hat{a}_\rho + \overset{1}{\cancel{\rho}}\,d\phi\,\hat{a}_\phi + \overset{0}{\cancel{dz}}\hat{a}_z)$$

$$+\int_{c_3}(\hat{a}_\rho + \overset{2}{\cancel{z}}\hat{a}_\phi + \hat{a}_z)\cdot(d\rho\,\hat{a}_\rho + \rho\, \overset{0}{\cancel{d\phi}}\hat{a}_\phi + \overset{0}{\cancel{dz}}\hat{a}_z)$$

$$= \cancel{\int_{\rho=0}^{1} d\rho} + \int_{\phi=0}^{\pi/2} z\, d\phi + \cancel{\int_{\rho=1}^{0} d\rho} = \left(\frac{\pi}{2} - 0\right) z$$

$$\oint_c \bar{F}\cdot d\bar{l} = \underline{\underline{\pi}} \quad \therefore$$