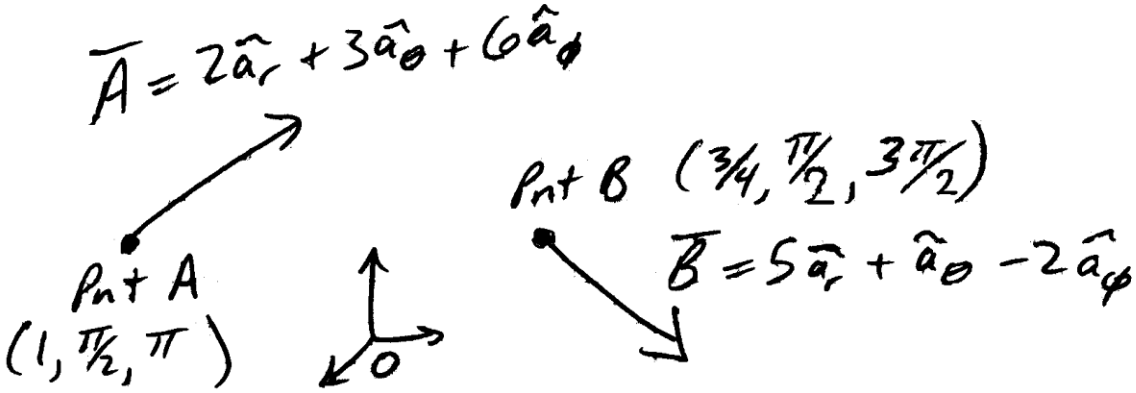
Example- Add the vectors $\bar{A} = 2\hat{a}_r + 3\hat{a}_\theta + 6\hat{a}_\phi$ and $\bar{B} = 5\hat{a}_r + \hat{a}_\theta - 2\hat{a}_\phi$ defined in spherical coordinates at the points $A(1, \pi/2, \pi)$ and $B(3/4, \pi/2, 3\pi/2)$, respectively.



when $\bar{A}$ & $\bar{B}$ are <u>NOT</u> defined at the same point

$$\bar{A} + \bar{B} \neq (A_r + B_r)\hat{a}_r + (A_\theta + B_\theta)\hat{a}_\theta + (A_\phi + B_\phi)\hat{a}_\phi$$

$$\bar{A} + \bar{B} \neq (2+5)\hat{a}_r + (3+1)\hat{a}_\theta + (6-2)\hat{a}_\phi$$

$$\neq 7\hat{a}_r + 4\hat{a}_\theta + 4\hat{a}_\phi$$

since $\hat{a}_r$, $\hat{a}_\theta$, & $\hat{a}_\phi$ point different directions at A & B

$\Rightarrow$ easiest solution convert $\hat{a}_r$, $\hat{a}_\theta$, & $\hat{a}_\phi$ to $\hat{a}_x$, $\hat{a}_y$, & $\hat{a}_z$

<u>OR</u>

calculate A_x, A_y, & A_z from A_r, A_θ, & A_ϕ

then add $\bar{A}$ + $\bar{B}$ together

ex. cont.

Method 1 (convert unit vectors)

@ pnt $A\left(\overset{r}{1}, \overset{\theta}{\pi/2}, \overset{\phi}{\pi}\right)$

$$\hat{a}_r = \cancelto{1}{\sin \tfrac{\pi}{2}} \cancelto{-1}{\cos \pi}\, \hat{a}_x + \sin \tfrac{\pi}{2} \cancelto{0}{\sin \pi}\, \hat{a}_y + \cancelto{0}{\cos \tfrac{\pi}{2}}\, \hat{a}_z$$

$$\underline{\hat{a}_r = -\hat{a}_x}$$

$$\hat{a}_\theta = \cancelto{0}{\cos \tfrac{\pi}{2}} \cos \pi\, \hat{a}_x + \cancelto{0}{\cos \tfrac{\pi}{2}} \sin \pi\, \hat{a}_y - \sin \tfrac{\pi}{2}\, \hat{a}_z$$

$$\underline{\hat{a}_\theta = -\hat{a}_z}$$

$$\hat{a}_\phi = -\cancelto{0}{\sin \pi}\, \hat{a}_x + \cancelto{-1}{\cos \pi}\, \hat{a}_y = \underline{-\hat{a}_y}$$

So

$$\bar{A} = 2\hat{a}_r + 3\hat{a}_\theta + 6\hat{a}_\phi = \underline{-2\hat{a}_x - 3\hat{a}_z - 6\hat{a}_y}$$

@ pnt $B\left(\overset{r}{3/4}, \overset{\theta}{\pi/2}, \overset{\phi}{3\pi/2}\right)$

$$\hat{a}_r = \sin \tfrac{\pi}{2} \cancelto{0}{\cos \tfrac{3\pi}{2}}\, \hat{a}_x + \sin \tfrac{\pi}{2} \sin \tfrac{3\pi}{2}\, \hat{a}_y + \cancelto{0}{\cos \tfrac{\pi}{2}}\, \hat{a}_z = \underline{-\hat{a}_y}$$

$$\hat{a}_\theta = \cos \tfrac{\pi}{2} \cancelto{0}{\cos \tfrac{3\pi}{2}}\, \hat{a}_x + \cancelto{0}{\cos \tfrac{\pi}{2}} \sin \tfrac{3\pi}{2}\, \hat{a}_y - \sin \tfrac{\pi}{2}\, \hat{a}_z = \underline{-\hat{a}_z}$$

$$\hat{a}_\phi = -\sin \tfrac{3\pi}{2}\, \hat{a}_x + \cos \tfrac{3\pi}{2}\, \hat{a}_y = \underline{\hat{a}_x}$$

So

$$\bar{B} = 5\hat{a}_r + \hat{a}_\theta - 2\hat{a}_\phi = \underline{-5\hat{a}_y - \hat{a}_z - 2\hat{a}_x}$$

$$\bar{A} + \bar{B} = (-2-2)\hat{a}_x + (-6-5)\hat{a}_y + (-3-1)\hat{a}_z$$

$$\underline{\underline{\bar{A} + \bar{B} = -4\hat{a}_x - 11\hat{a}_y - 4\hat{a}_z}}$$

ex. cont.

Method 2 (find $A_x, A_y, A_z, \ldots$)

@ pnt A $(r=1, \theta=\pi/2, \phi=\pi)$

$$A_x = 2\sin\tfrac{\pi}{2}\cos\pi + 3\underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{0}\cos\pi - 6\underbrace{\cancel{\sin\pi}}_{0} = -2$$

$$A_y = 2\sin\tfrac{\pi}{2}\underbrace{\cancel{\sin\pi}}_{0} + 3\cos\tfrac{\pi}{2}\underbrace{\cancel{\sin\pi}}_{0} + 6\cos\pi = -6$$

$$A_z = 2\cos\tfrac{\pi}{2} - 3\sin\tfrac{\pi}{2} = -3$$

so $\bar{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\underline{\bar{A} = -2\hat{a}_x - 6\hat{a}_y - 3\hat{a}_z}$

Similarly @ pnt B $(r=3/4, \theta=\pi/2, \phi=3\pi/2)$

$$B_x = 5\sin\tfrac{\pi}{2}\underbrace{\cancel{\cos\tfrac{3\pi}{2}}}_{0} + (1)\underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{0}\cos\tfrac{3\pi}{2} - (-2)\sin\tfrac{3\pi}{2} = -2$$

$$B_y = 5\sin\tfrac{\pi}{2}\sin\tfrac{3\pi}{2} + (1)\underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{0}\sin\tfrac{3\pi}{2} + (-2)\cos\tfrac{3\pi}{2} = -5$$

$$B_z = 5\underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{0} - (1)\sin\tfrac{\pi}{2} = -1$$

So $\bar{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$

$\bar{B} = -2\hat{a}_x - 5\hat{a}_y - \hat{a}_z$

and

$$\bar{A} + \bar{B} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$$

$\underline{\bar{A} + \bar{B} = -4\hat{a}_x - 11\hat{a}_y - 4\hat{a}_z}$ $\therefore$ Same!