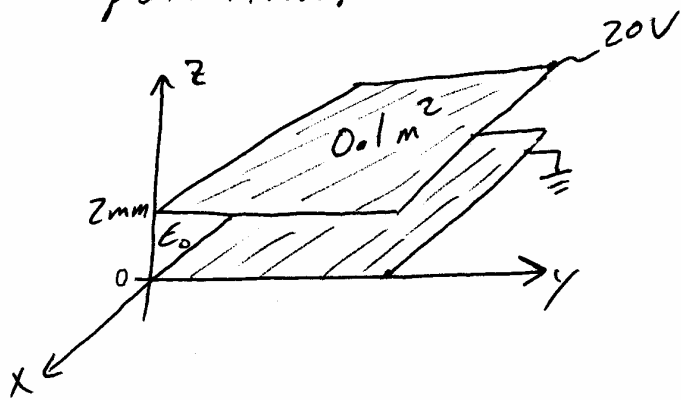


ex. Find the energy density and stored electric energy in a parallel-plate capacitor (see picture) with an air-dielectric and 20V potential.



$$\vec{E} = -\hat{a}_z \frac{V}{d} = -\hat{a}_z \frac{20}{2 \times 10^{-3}}$$

$$\vec{E} = \begin{cases} -\hat{a}_z 10,000 \text{ V/m} & \text{between plates} \\ 0 & \text{elsewhere} \end{cases}$$

Energy density

$$w_e = \frac{1}{2} \epsilon_0 |\vec{E}|^2 = \left(\frac{1}{2}\right) 8.854 \times 10^{-12} (10,000)^2$$

$$w_e = 4.427 \times 10^{-4} \text{ J/m}^3 = 442.7 \text{ } \mu\text{J/m}^3 \quad \text{between plates}$$

Energy Stored

$$W_e = \frac{1}{2} \int_V \epsilon_0 |\vec{E}|^2 dV = \int_V w_e dV$$

$$= 4.427 \times 10^{-4} \int_V dx dy dz \quad \text{volume between plates}$$

$$= 4.427 \times 10^{-4} [(0.1)(2 \times 10^{-3})]$$

$$W_e = 8.854 \times 10^{-8} \text{ J} = 88.54 \text{ nJ}$$

Check $C = \frac{\epsilon_0 S}{d} = \frac{8.854 \times 10^{-12} (0.1)}{2 \times 10^{-3}} = 0.4427 \text{ nF}$

$$W_e = \frac{1}{2} C V^2 = \frac{1}{2} (0.4427 \times 10^{-9}) (20)^2 = 88.54 \text{ nJ}$$

→ Same!