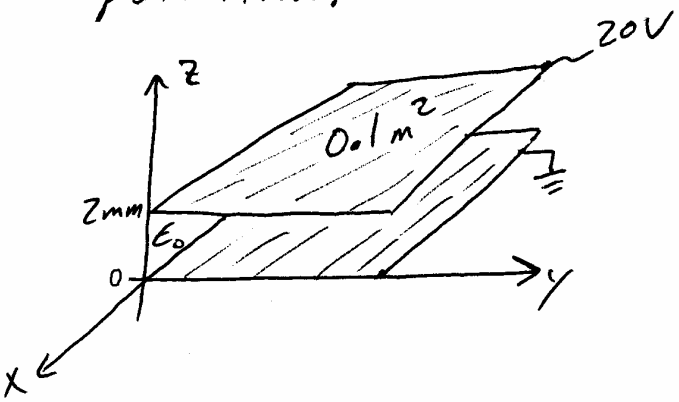
ex. Find the energy density and stored electric energy in a parallel-plate capacitor (see picture) with an air-dielectric and 20V potential.



$$\bar{E} = -\hat{a}_z \frac{V}{d} = -\hat{a}_z \frac{20}{2\times10^{-3}}$$

$$\bar{E} = \begin{cases} -\hat{a}_z\, 10{,}000\ V/m & \text{between plates} \\ 0 & \text{elsewhere} \end{cases}$$

Energy density

$$w_e = \tfrac{1}{2}\epsilon_0 |\bar{E}|^2 = \left(\tfrac{1}{2}\right) 8.854\times10^{-12} (10{,}000)^2$$

$$w_e = 4.427\times10^{-4}\ J/m^3 = 442.7\ \mu J/m^3 \quad \text{between plates}$$

Energy Stored

$$W_e = \tfrac{1}{2}\int_V \epsilon_0 |\bar{E}|^2 dV = \int_V w_e\, dV$$

$$= 4.427\times10^{-4} \int_V dx\,dy\,dz \quad \text{volume between plates}$$

$$= 4.427\times10^{-4}\left[(0.1)(2\times10^{-3})\right]$$

$$W_e = 8.854\times10^{-8}\ J = 88.54\ nJ$$

Check $C = \dfrac{\epsilon_0 S}{d} = \dfrac{8.854\times10^{-12}(0.1)}{2\times10^{-3}} = 0.4427\ nF$

$$W_e = \tfrac{1}{2} C V^2 = \tfrac{1}{2}(0.4427\times10^{-9})(20)^2 = 88.54\ nJ$$

Same!