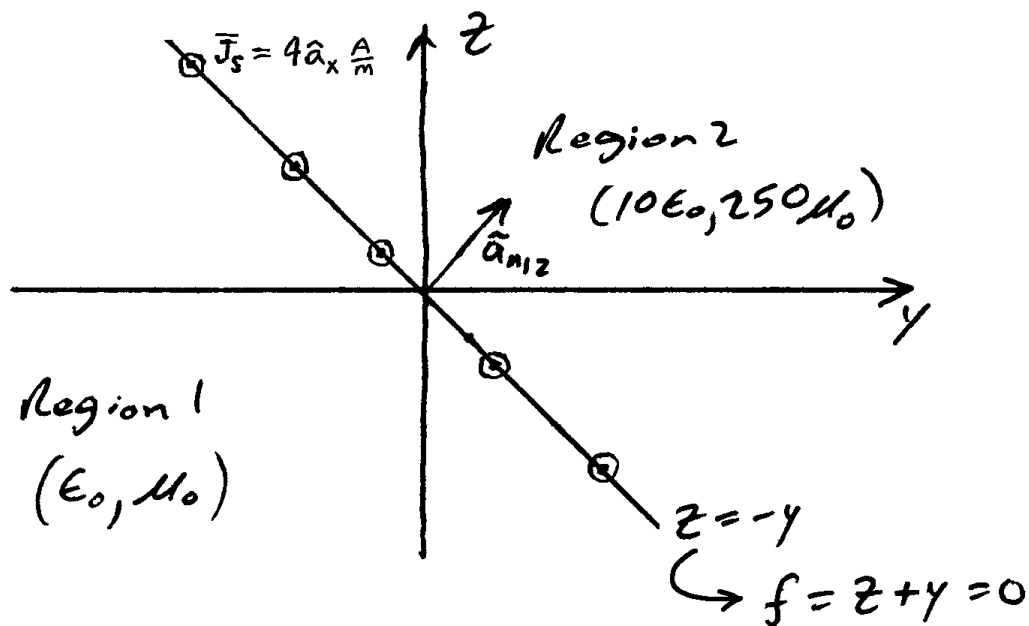


**Example-** Region 1 (air-  $\epsilon_1 = \epsilon_0$ ,  $\mu_1 = \mu_0$ ) is separated from region 2, composed of a ferrite material ( $\epsilon_2 = 10\epsilon_0$ ,  $\mu_2 = 250\mu_0$ ), by the planar interface  $z + y = 0$  shown. On the interface, there is a surface current density  $\bar{J}_s = 4\hat{a}_x$  (A/m). If the magnetic flux density vector near the interface in region 2 is  $\bar{B}_2 = \mu_0(750\hat{a}_x - 1000\hat{a}_y + 500\hat{a}_z)$  T, find the magnetic field  $\bar{H}_1$  in region 1.

Side View ( $x=0$  plane) +  $x$  out of page



First, find surface normal from region 1 into 2.

By inspection,  $\bar{A} = 1\hat{a}_y + 1\hat{a}_z$  is normal to interface

$$\hat{a}_{n12} = \frac{\bar{A}}{|\bar{A}|} = \frac{\hat{a}_y + \hat{a}_z}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} (\hat{a}_y + \hat{a}_z)$$

Or, use gradient

$$\hat{a}_{n12} = \frac{\bar{\nabla}f}{|\bar{\nabla}f|} = \frac{\bar{\nabla}(y+z)}{|\bar{\nabla}(y+z)|} = \frac{\hat{a}_y + \hat{a}_z}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} (\hat{a}_y + \hat{a}_z)$$

Apply tangential boundary condition

$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_{n12} = \bar{J}_s \quad \begin{array}{c} \nearrow x \\ \searrow y \\ z \end{array}$$

assume  $\bar{H}_1 = H_{1x}\hat{a}_x + H_{1y}\hat{a}_y + H_{1z}\hat{a}_z$

$$\bar{H}_2 = \frac{\bar{B}_2}{\mu_2} = \frac{\mu_0 (750\hat{a}_x - 1000\hat{a}_y + 500\hat{a}_z)}{250\mu_0}$$

$$= 3\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z \text{ (A/m)}$$

$$\left[ (H_{1x} - 3)\hat{a}_x + (H_{1y} + 4)\hat{a}_y + (H_{1z} - 2)\hat{a}_z \right] \times \frac{1}{\sqrt{2}} (\hat{a}_y + \hat{a}_z) = 4\hat{a}_x$$

$$\left[ \left( \frac{H_{1x} - 3}{\sqrt{2}} \right) \hat{a}_z - \left( \frac{H_{1x} - 3}{\sqrt{2}} \right) \hat{a}_y \right] + \left[ 0 + \left( \frac{H_{1y} + 4}{\sqrt{2}} \right) \right] \hat{a}_x$$

$$+ \left[ \left( \frac{H_{1z} - 2}{\sqrt{2}} \right) (-\hat{a}_x) + 0 \right] = 4\hat{a}_x$$

$\Rightarrow$  Equate vector components

$$\hat{a}_z \text{ term(s)} \quad \frac{H_{1x} - 3}{\sqrt{2}} = 0 \quad \Rightarrow \quad H_{1x} = 3 \text{ A/m}$$

$$\hat{a}_y \text{ term(s)} \quad - \left( \frac{H_{1x} - 3}{\sqrt{2}} \right) = 0 \quad \Rightarrow \quad \underline{B_{1x} = \mu_0 H_{1x} = 3\mu_0 \text{ T}}$$

$$\hat{a}_x \text{ term(s)} \quad \frac{H_{1y} + 4}{\sqrt{2}} - \left( \frac{H_{1z} - 2}{\sqrt{2}} \right) = 4$$

$$\hookrightarrow \underline{H_{1y} - H_{1z} = 4\sqrt{2} - 4 - 2} \quad \textcircled{A}$$

## Apply Normal Boundary Condition

$$\hat{a}_{n12} \cdot (\bar{B}_1 - \bar{B}_2) = 0 \quad \text{where } \bar{B}_1 = \mu_1 \bar{H}_1 = \mu_0 \bar{H}_1 \\ + \bar{B}_2 = \mu_2 \bar{H}_2$$

$$\left( \frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}} \right) \cdot \left[ (3\mu_0 \hat{a}_x + \mu_0 H_{1y} \hat{a}_y + \mu_0 H_{1z} \hat{a}_z) \right. \\ \left. - \mu_0 (750 \hat{a}_x - 1000 \hat{a}_y + 500 \hat{a}_z) \right] = 0$$

$$\left( \frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}} \right) \cdot \left[ (3\mu_0 - 750\mu_0) \hat{a}_x + (H_{1y} \mu_0 + 1000 \mu_0) \hat{a}_y \right. \\ \left. + (H_{1z} \mu_0 - 500 \mu_0) \hat{a}_z \right] = 0$$

$$\left( 0 + \frac{H_{1y} \mu_0 + 1000 \mu_0}{\sqrt{2}} + 0 \right) + \left( 0 + 0 + \frac{H_{1z} \mu_0 - 500 \mu_0}{\sqrt{2}} \right) = 0$$

$$\hookrightarrow \underline{H_{1y} + H_{1z} = -500} \quad \textcircled{B}$$

→ Two - eqns - 2 unknowns

$$\textcircled{A} + \textcircled{B} \quad 2H_{1y} + 0 = 4\sqrt{2} - 4 - 2 + (-500)$$

$$\underline{H_{1y} = -250.171573 \text{ A/m}}$$

$$\underline{H_{1z} = -500 - H_{1y} = -249.82843 \text{ A/m}}$$

$$\underline{\underline{\bar{H}_1 = 3\hat{a}_x - 250.1716 \hat{a}_y - 249.828 \hat{a}_z \text{ A/m} \quad \therefore}}$$