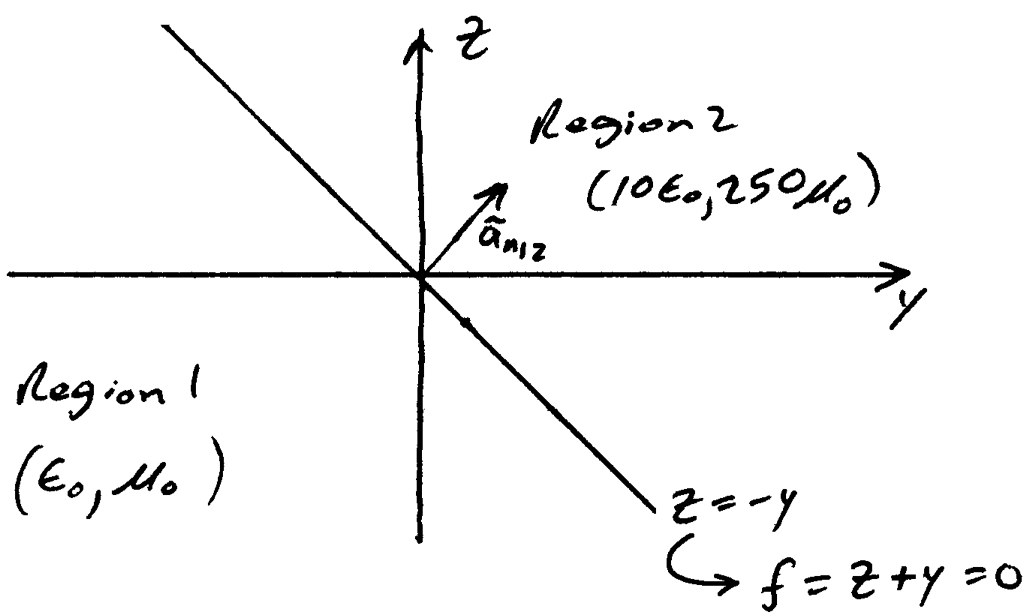
Example- Region 1 (air- $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$) is separated from region 2, composed of a ferrite material ($\varepsilon_2 = 10\varepsilon_0$, $\mu_2 = 250\mu_0$), by the planar interface $z + y = 0$ shown. On the interface, there is a surface current density $\bar{J}_S = 4\hat{a}_x$ (A/m). If the magnetic flux density vector near the interface in region 2 is $\bar{B}_2 = \mu_0(750\hat{a}_x - 1000\hat{a}_y + 500\hat{a}_z)$ T, find the magnetic field $\bar{H}_1$ in region 1.

Side View (x=0 plane) +x out of page



First, find surface normal from region 1 into 2.

By inspection, $\bar{A} = 1\hat{a}_y + 1\hat{a}_z$ is normal to interface

$$\hat{a}_{n12} = \frac{\bar{A}}{|\bar{A}|} = \frac{\hat{a}_y + \hat{a}_z}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}(\hat{a}_y + \hat{a}_z)$$

Or, use gradient

$$\hat{a}_{n12} = \frac{\bar{\nabla} f}{|\bar{\nabla} f|} = \frac{\bar{\nabla}(y+z)}{|\bar{\nabla}(y+z)|} = \frac{\hat{a}_y + \hat{a}_z}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}(\hat{a}_y + \hat{a}_z)$$

Apply tangential boundary condition

$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_{n12} = \bar{J}_s$$

$$x \to y \to z$$

assume $\bar{H}_1 = H_{1x}\hat{a}_x + H_{1y}\hat{a}_y + H_{1z}\hat{a}_z$

$$\bar{H}_2 = \bar{B}_2 / \mu_2 = \frac{\mu_0(750\hat{a}_x - 1000\hat{a}_y + 500\hat{a}_z)}{250\mu_0}$$

$$= 3\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z \text{ (A/m)}$$

$$\left[(H_{1x}-3)\hat{a}_x + (H_{1y}+4)\hat{a}_y + (H_{1z}-2)\hat{a}_z\right] \times \frac{1}{\sqrt{2}}(\hat{a}_y + \hat{a}_z) = 4\hat{a}_x$$

$$\left[\left(\frac{H_{1x}-3}{\sqrt{2}}\right)\hat{a}_z - \left(\frac{H_{1x}-3}{\sqrt{2}}\right)\hat{a}_y\right] + \left[0 + \left(\frac{H_{1y}+4}{\sqrt{2}}\right)\right]\hat{a}_x$$

$$+\left[\left(\frac{H_{1z}-2}{\sqrt{2}}\right)(-\hat{a}_x) + 0\right] = 4\hat{a}_x$$

$\Rightarrow$ Equate vector components

$\hat{a}_z$ term(s) $\quad \dfrac{H_{1x}-3}{\sqrt{2}} = 0 \Rightarrow \underline{H_{1x} = 3 \text{ A/m}}$

$$\Downarrow$$

$\hat{a}_y$ term(s) $\quad -\left(\dfrac{H_{1x}-3}{\sqrt{2}}\right) = 0 \Rightarrow \underline{B_{1x} = \mu_0 H_{1x} = 3\mu_0 \text{ T}}$

$\hat{a}_x$ term(s) $\quad \dfrac{H_{1y}+4}{\sqrt{2}} - \left(\dfrac{H_{1z}-2}{\sqrt{2}}\right) = 4$

$$\hookrightarrow \underline{H_{1y} - H_{1z} = 4\sqrt{2} - 4 - 2} \quad (A)$$

Apply Normal Boundary Condition

$$\hat{a}_{n12} \cdot \left(\bar{B}_1 - \bar{B}_2\right) = 0 \quad \text{where } \bar{B}_1 = \mu_1 \bar{H}_1 = \mu_0 \bar{H}_1$$
$$\& \ \bar{B}_2 = \mu_2 \bar{H}_2$$

$$\left(\frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}}\right) \cdot \left[\left(3\mu_0 \hat{a}_x + \mu_0 H_{1y} \hat{a}_y + \mu_0 H_{1z} \hat{a}_z\right) - \mu_0\left(750\hat{a}_x - 1000\hat{a}_y + 500\hat{a}_z\right)\right] = 0$$

$$\left(\frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}}\right) \cdot \left[\left(3\mu_0 - 750\mu_0\right)\hat{a}_x + \left(H_{1y}\mu_0 + 1000\mu_0\right)\hat{a}_y + \left(H_{1z}\mu_0 - 500\mu_0\right)\hat{a}_z\right] = 0$$

$$\left(0 + \frac{H_{1y}\mu_0 + 1000\mu_0}{\sqrt{2}} + 0\right) + \left(0 + 0 + \frac{H_{1z}\mu_0 - 500\mu_0}{\sqrt{2}}\right) = 0$$

$$\hookrightarrow \underline{H_{1y} + H_{1z} = -500} \quad (B)$$

$\rightarrow$ Two-eq'ns - 2 unknowns

$$(A) + (B) \qquad 2H_{1y} + 0 = 4\sqrt{2} - 4 - 2 + (-500)$$

$$\underline{H_{1y} = -250.171573 \ A/m}$$

$$\underline{H_{1z} = -500 - H_{1y} = -249.82843 \ A/m}$$

$$\underline{\underline{\bar{H}_1 = 3\hat{a}_x - 250.1716\hat{a}_y - 249.828\hat{a}_z \ A/m}} \quad \therefore$$