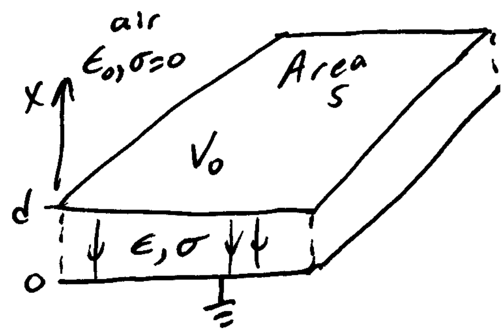
ex. Lossy Parallel Plate Capacitor Resistance



Assume $d \ll$ plate dimensions

$\Rightarrow$ No fringing of $\bar{E}$, $\bar{D}$, & $\bar{J}$ all entirely in lossy dielectric

$\Rightarrow \rho_v = 0$ (no extra free charges)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \underbrace{\frac{\partial^2 V}{\partial y^2}}_{\to 0} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{\to 0} = 0$$

$$\int \frac{d^2 V}{dx^2}\, dx = \int 0\, dx \rightarrow \frac{dV}{dx} = A$$

$$\int \frac{dV}{dx}\, dx = \int dV = \int A\, dx$$

$$V = Ax + B \quad \leftarrow \text{general solution}$$

Apply boundary conditions

$$V(x=0) = 0 = A(0) + B \quad \Rightarrow B = 0 \text{ \& } V = Ax$$

$$V(x=d) = V_0 = Ad \quad \Rightarrow A = V_0/d$$

$$V(x) = \frac{V_0}{d} x \qquad 0 \le x \le d$$

$$\bar{E} = -\bar{\nabla} V = -\hat{a}_x \frac{dV}{dx} = -\hat{a}_x \frac{V_0}{d} \quad 0 \le x \le d \text{ in dielectric}$$

$$\bar{D} = \epsilon \bar{E} = -\hat{a}_x \frac{\epsilon V_0}{d} \quad 0 \le x \le d \text{ in dielectric}$$

$$\bar{J} = \sigma \bar{E} = -\hat{a}_x \frac{\sigma V_0}{d} \quad 0 \le x \le d \text{ in dielectric}$$

Find Resistance

use $R = V/I$ where $V = V_0$ (given)

$I = \oint\!\!\oint_S \bar{J} \cdot d\bar{s}$ choose S to be conformal box around top plate

$$= \iint_{Top} \bar{J}^{\,0} \cdot +d\bar{s}_z + \iint_{sides} \bar{J}^{\,0} \cdot \pm d\bar{s}_{y,z} + \iint_{Bottom} -\hat{a}_x \frac{\sigma V_0}{d} \cdot -\hat{a}_x \, dy\, dz$$

$$= \frac{\sigma V_0}{d} \underbrace{\iint_{Bottom} dy\, dz}_{S} \Rightarrow I = \frac{\sigma V_0 S}{d}$$

$$R = \frac{V_0}{\left(\frac{\sigma V_0 S}{d}\right)} = \underline{\underline{\frac{d}{\sigma S}}}$$ ← agrees w/ $R = \frac{\ell}{\sigma A}$

ex. Find R + C for a parallel plate capacitor with 20 mm x 20 mm plates separated by a 0.1 mm thick slab of Bakelite ($\epsilon_r = 5, \sigma = 10^{-10}$ S/m)

$$R = \frac{d}{\sigma S} = \frac{0.1 \times 10^{-3}}{10^{-10}(20 \times 10^{-3})^2} \Rightarrow \underline{\underline{R = 2.5 \times 10^9 \,\Omega = 2.5\, G\Omega}}$$

$$C = \frac{\epsilon S}{d} = \frac{5(8.8541878 \times 10^{-12})(20 \times 10^{-3})^2}{0.1 \times 10^{-3}}$$

$$\underline{\underline{C = 1.77084 \times 10^{-10}\, F = 0.177\, nF = 177.1\, pF}}$$

If $V = 10\,V$, the force between the plates is

$$|F| = \frac{V_0^2 \epsilon S}{2d^2} = \frac{10^2(5)8.8542 \times 10^{-12}(0.02)^2}{2(0.1 \times 10^{-3})^2} = \underline{\underline{88.54\, \mu N}}$$