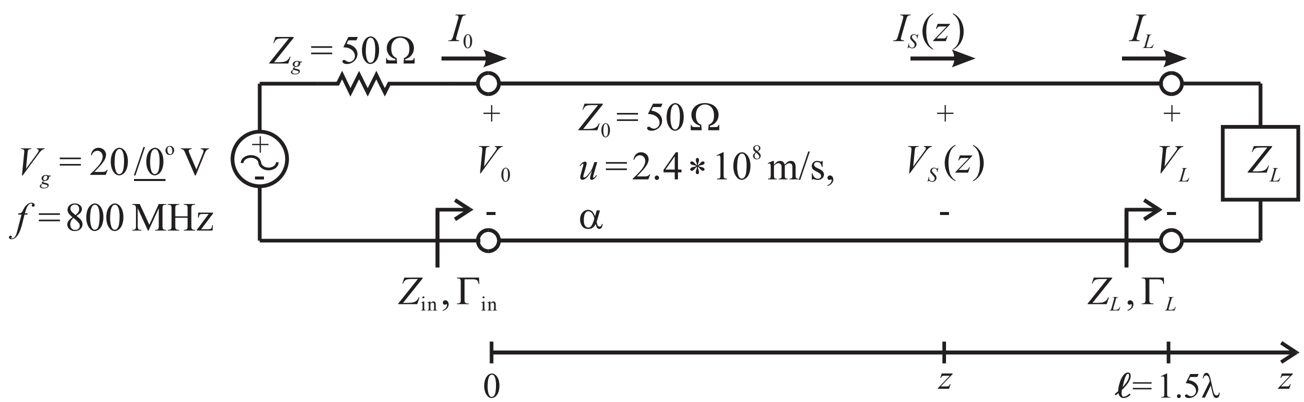
Lossy & Lossless transmission line (TL) standing wave examples

For the TL circuit shown below, we will study the current & voltage standing waves for both the lossless ($\alpha = 0$) and lossy ($\alpha = 0.4$ Np/m) cases for loads Z_L: $Z_{sc} = 0$ (short circuit), $Z_{oc} = \infty$ (open circuit), $Z_{gen} = 30 + j50\ \Omega$ (general), and $Z_m = Z_0 = 50\ \Omega$ (matched). As is often done in TL datasheets, the characteristic impedance has been rounded off to a real number.



Given values

$Vg := 20\cdot e^{j\cdot 0}$ V $Zg := 50$ Ω $f := 800\cdot 10^6$ Hz

$Z0 := 50$ Ω $u := 2.4\cdot 10^8$ m/s $\alpha := 0.4$ Np/m $L\lambda := 1.5$

$Zsc := 0$ Ω $Zoc := 10^{99}$ Ω $Zgen := 30 + j\cdot 50$ Ω $Zm := Z0$

General quantities (load independent)

$\omega := 2\cdot\pi\cdot f$ $\omega = 5.027\times 10^9$ rad/s $\lambda := \dfrac{u}{f}$ $\lambda = 0.3$ m

$L := L\lambda\cdot\lambda$ $L = 0.45$ m

$\beta := \dfrac{\omega}{u}$ $\beta = 20.944$ rad/m (used by itself for lossless cases)

$\gamma := \alpha + j\cdot\beta$ $\gamma = 0.4 + 20.944i$ 1/m (used for lossy cases)

$n := 0 .. 150$ $z_n := \dfrac{n}{150}\cdot L$ define locations along the TL

Short circuit load, Lossless TL ($\alpha = 0$)

$$\Gamma Lsc := \frac{Zsc - Z0}{Zsc + Z0} \qquad \boxed{\Gamma Lsc = -1} \qquad \boxed{SWRsc = \infty}$$

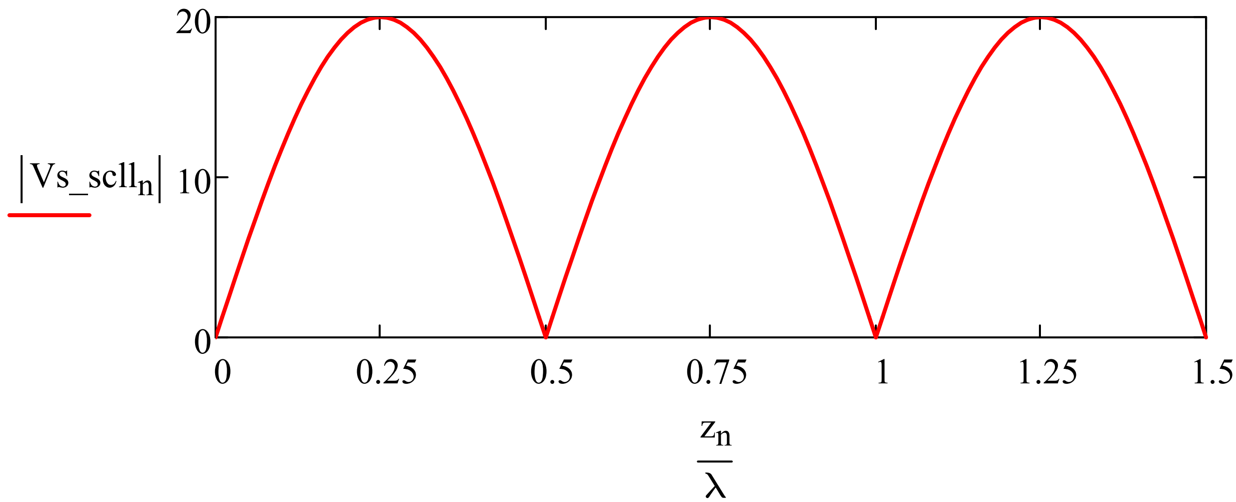
$$\Gamma inscll := \Gamma Lsc \cdot e^{-j \cdot 2 \cdot \beta \cdot L} \qquad Zinscll := Z0 \cdot \frac{1 + \Gamma inscll}{1 - \Gamma inscll}$$

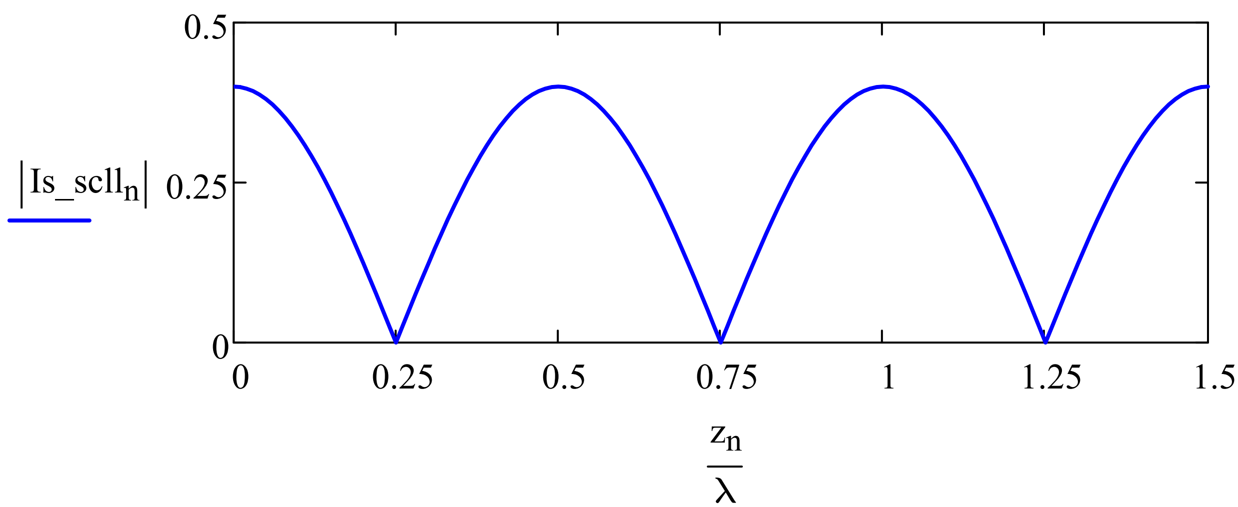
$$V0scll := Vg \cdot \frac{Zinscll}{Zg + Zinscll} \qquad I0scll := \frac{Vg}{Zg + Zinscll}$$

$$V0p\_scll := 0.5 \cdot (V0scll + Z0 \cdot I0scll) \qquad V0m\_scll := V0scll - V0p\_scll$$

$$Vs\_scll_n := V0p\_scll \cdot e^{-j \cdot \beta \cdot z_n} + V0m\_scll \cdot e^{j \cdot \beta \cdot z_n}$$

$$Is\_scll_n := \frac{V0p\_scll}{Z0} \cdot e^{-j \cdot \beta \cdot z_n} - \frac{V0m\_scll}{Z0} \cdot e^{j \cdot \beta \cdot z_n}$$





Note, the current & voltage standing waves repeat at $\lambda/2$ intervals!

Short circuit load, Lossy TL

$\Gamma insc := \Gamma Lsc \cdot e^{-2\cdot\gamma\cdot L}$

$Zinsc := Z0 \cdot \dfrac{1+\Gamma insc}{1-\Gamma insc}$

$V0sc := Vg \cdot \dfrac{Zinsc}{Zg + Zinsc}$

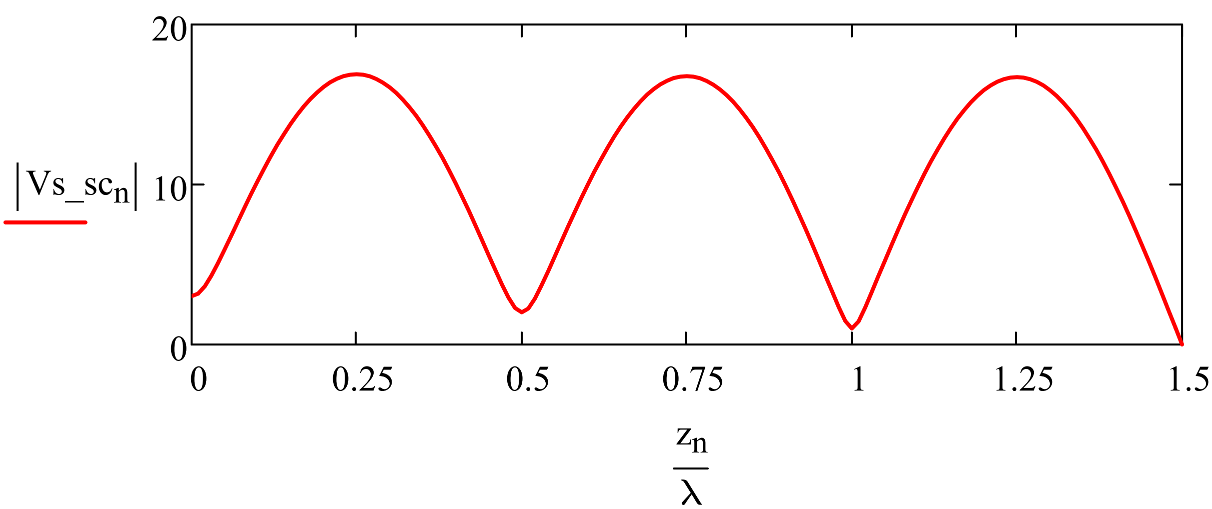
$I0sc := \dfrac{Vg}{Zg + Zinsc}$

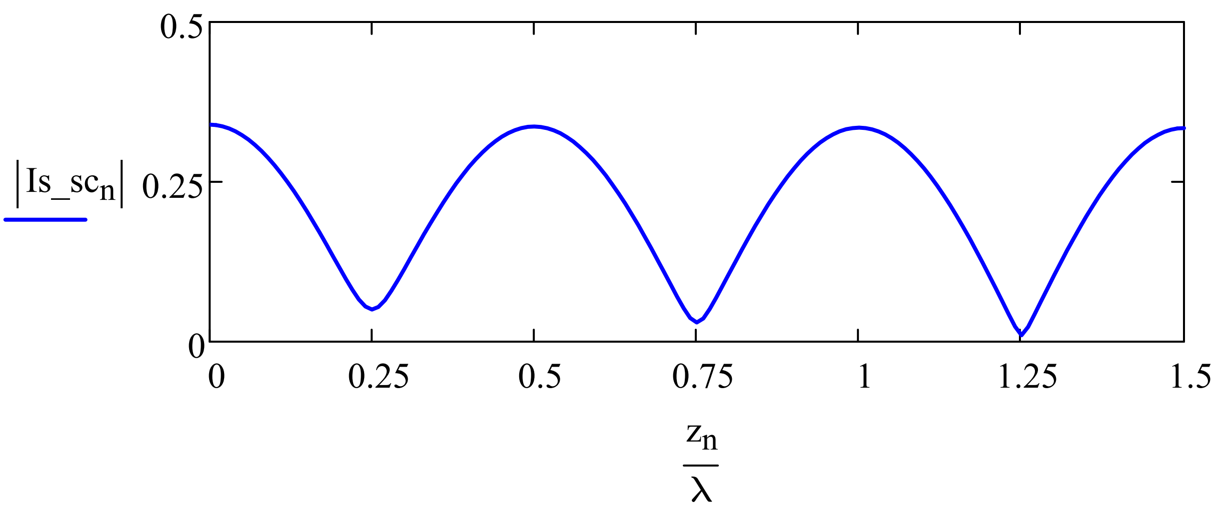
$V0p\_sc := 0.5\cdot(V0sc + Z0\cdot I0sc)$

$V0m\_sc := V0sc - V0p\_sc$

$$Vs\_sc_n := V0p\_sc \cdot e^{-\gamma\cdot z_n} + V0m\_sc \cdot e^{\gamma\cdot z_n}$$

$$Is\_sc_n := \frac{V0p\_sc}{Z0}\cdot e^{-\gamma\cdot z_n} - \frac{V0m\_sc}{Z0}\cdot e^{\gamma\cdot z_n}$$





Note how the standing wave minima are NOT zero as you go away from the load on the lossy TL. Max & min occur at λ/2 intervals.

Open circuit load, Lossless TL ($\alpha = 0$)

$$\Gamma Loc := \frac{Zoc - Z0}{Zoc + Z0} \qquad \boxed{\Gamma Loc = 1} \qquad \boxed{SWRoc = \infty}$$

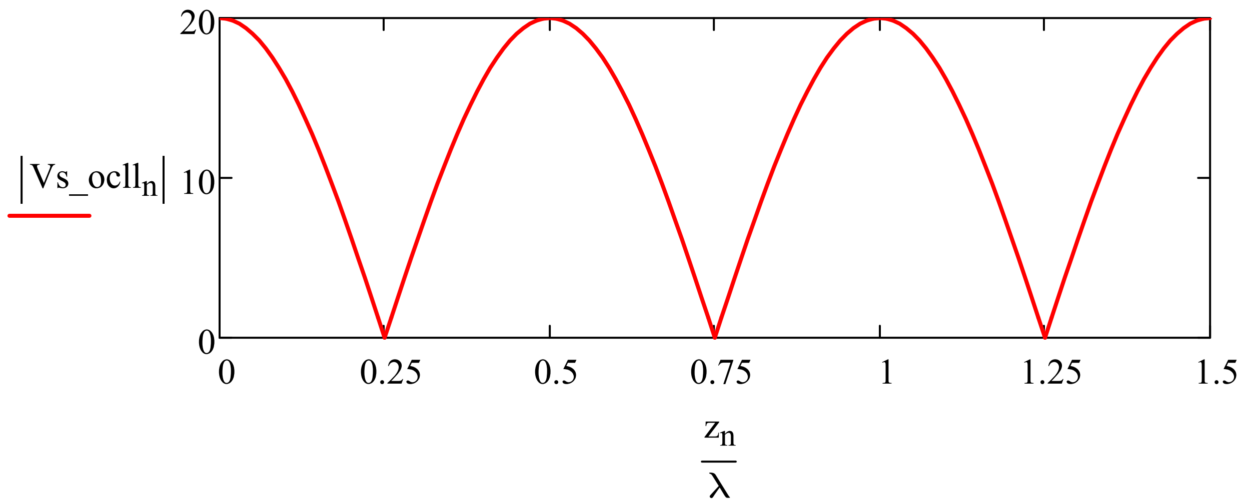
$$\Gamma inocll := \Gamma Loc \cdot e^{-j \cdot 2 \cdot \beta \cdot L} \qquad Zinocll := Z0 \cdot \frac{1 + \Gamma inocll}{1 - \Gamma inocll}$$

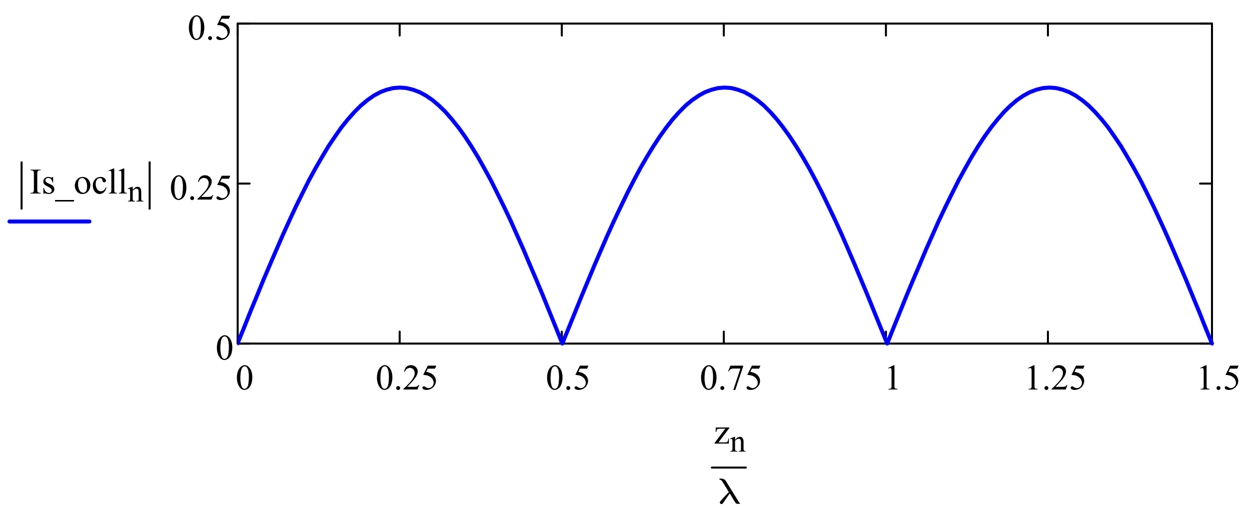
$$V0ocll := Vg \cdot \frac{Zinocll}{Zg + Zinocll} \qquad I0ocll := \frac{Vg}{Zg + Zinocll}$$

$$V0p\_ocll := 0.5 \cdot (V0ocll + Z0 \cdot I0ocll) \qquad V0m\_ocll := V0ocll - V0p\_ocll$$

$$Vs\_ocll_n := V0p\_ocll \cdot e^{-j \cdot \beta \cdot z_n} + V0m\_ocll \cdot e^{j \cdot \beta \cdot z_n}$$

$$Is\_ocll_n := \frac{V0p\_ocll}{Z0} \cdot e^{-j \cdot \beta \cdot z_n} - \frac{V0m\_ocll}{Z0} \cdot e^{j \cdot \beta \cdot z_n}$$





Note, the current & voltage standing waves repeat at $\lambda/2$ intervals!

Open circuit load, Lossy TL

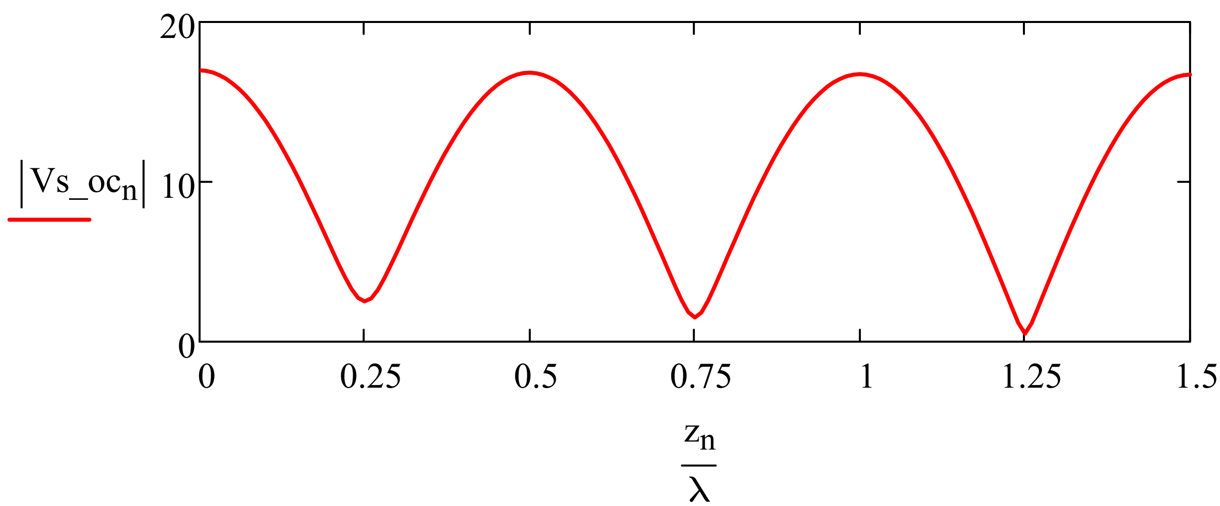
$$\Gamma inoc := \Gamma Loc \cdot e^{-2\cdot\gamma\cdot L} \qquad Zinoc := Z0\cdot\frac{1+\Gamma inoc}{1-\Gamma inoc}$$

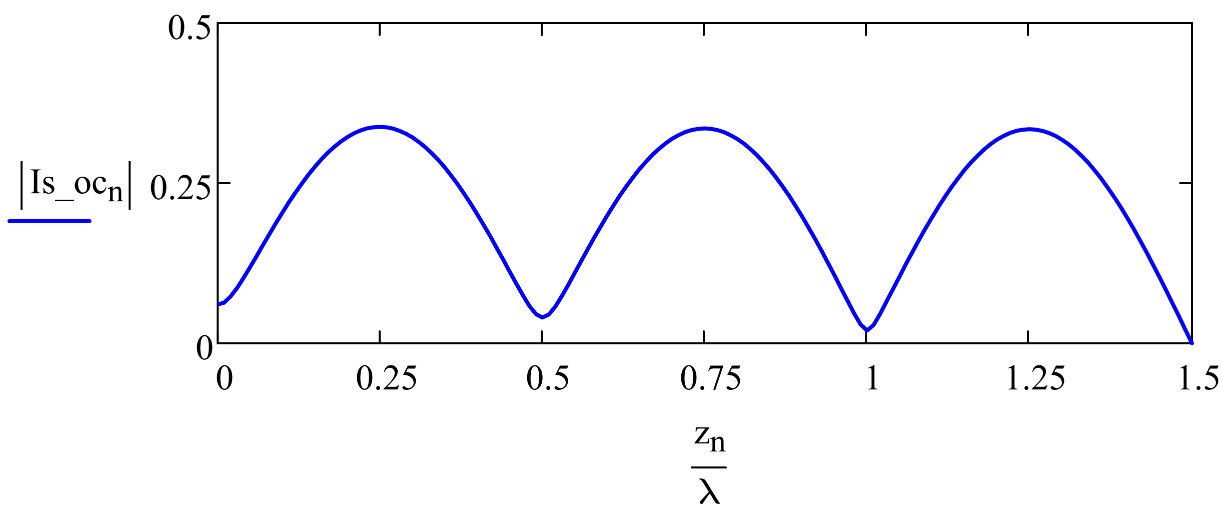
$$V0oc := Vg\cdot\frac{Zinoc}{Zg+Zinoc} \qquad I0oc := \frac{Vg}{Zg+Zinoc}$$

$$V0p\_oc := 0.5\cdot(V0oc + Z0\cdot I0oc) \qquad V0m\_oc := V0oc - V0p\_oc$$

$$Vs\_oc_n := V0p\_oc\cdot e^{-\gamma\cdot z_n} + V0m\_oc\cdot e^{\gamma\cdot z_n}$$

$$Is\_oc_n := \frac{V0p\_oc}{Z0}\cdot e^{-\gamma\cdot z_n} - \frac{V0m\_oc}{Z0}\cdot e^{\gamma\cdot z_n}$$





Note how the standing wave minima are NOT zero as you go away from the load on the lossy TL. Max & min occur at λ/2 intervals.

General load, Lossless TL ($\alpha = 0$)

$$\Gamma Lgen := \frac{Zgen - Z0}{Zgen + Z0} \qquad \boxed{|\Gamma Lgen| = 0.571}$$

$$Sgen := \frac{1 + |\Gamma Lgen|}{1 - |\Gamma Lgen|} \qquad \boxed{Sgen = 3.66}$$

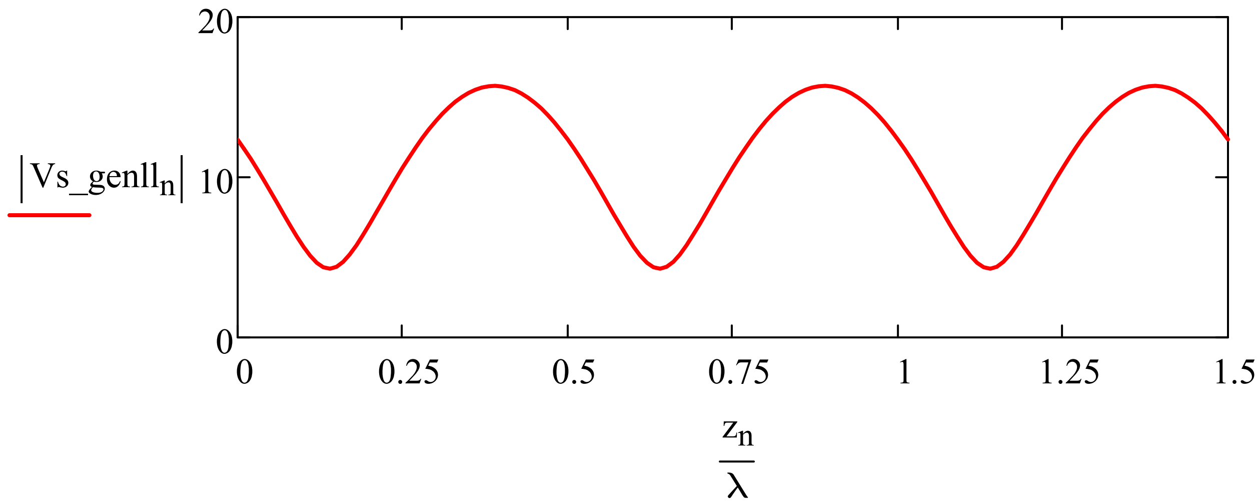
$$\Gamma ingenll := \Gamma Lgen \cdot e^{-j \cdot 2 \cdot \beta \cdot L} \qquad Zingenll := Z0 \cdot \frac{1 + \Gamma ingenll}{1 - \Gamma ingenll}$$

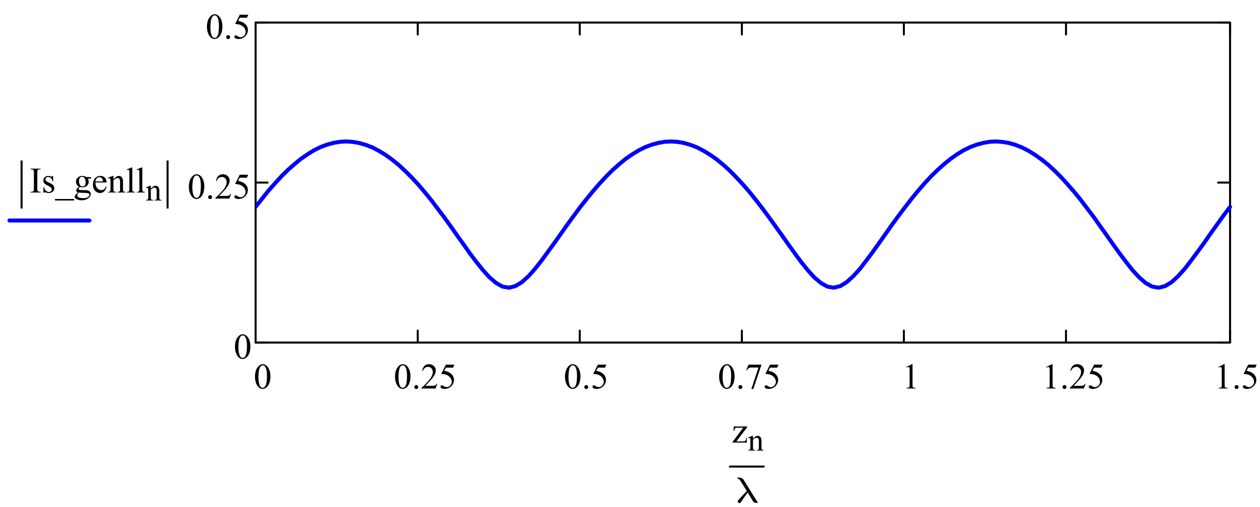
$$V0gll := Vg \cdot \frac{Zingenll}{Zg + Zingenll} \qquad I0gll := \frac{Vg}{Zg + Zingenll}$$

$$V0p\_genll := 0.5 \cdot (V0gll + Z0 \cdot I0gll) \qquad V0m\_genll := V0gll - V0p\_genll$$

$$Vs\_genll_n := V0p\_genll \cdot e^{-j \cdot \beta \cdot z_n} + V0m\_genll \cdot e^{j \cdot \beta \cdot z_n}$$

$$Is\_genll_n := \frac{V0p\_genll}{Z0} \cdot e^{-j \cdot \beta \cdot z_n} - \frac{V0m\_genll}{Z0} \cdot e^{j \cdot \beta \cdot z_n}$$





General load, Lossy TL

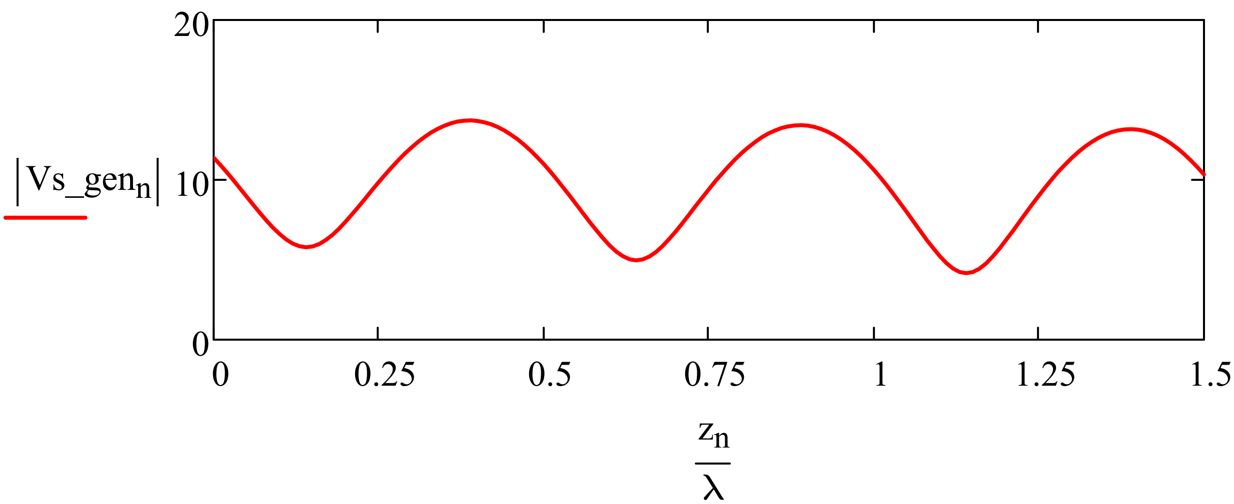
$$\Gamma ingen := \Gamma Lgen \cdot e^{-2\cdot\gamma\cdot L} \qquad Zingen := Z0\cdot\frac{1+\Gamma ingen}{1-\Gamma ingen}$$

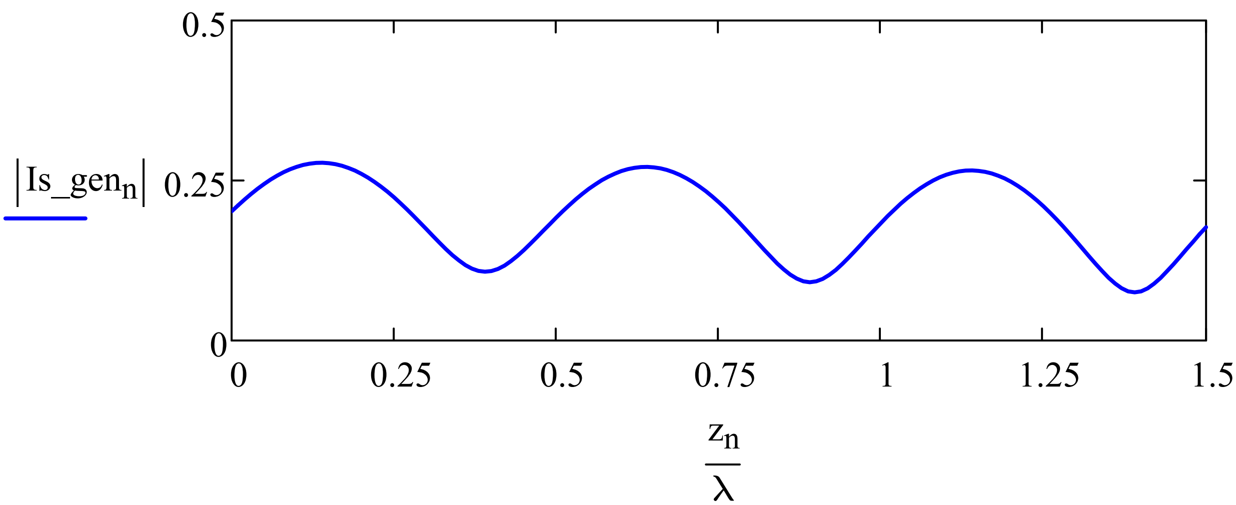
$$V0gen := Vg\cdot\frac{Zingen}{Zg+Zingen} \qquad I0gen := \frac{Vg}{Zg+Zingen}$$

$$V0p\_gen := 0.5\cdot(V0gen + Z0\cdot I0gen) \qquad V0m\_gen := V0gen - V0p\_gen$$

$$Vs\_gen_n := V0p\_gen\cdot e^{-\gamma\cdot z_n} + V0m\_gen\cdot e^{\gamma\cdot z_n}$$

$$Is\_gen_n := \frac{V0p\_gen}{Z0}\cdot e^{-\gamma\cdot z_n} - \frac{V0m\_gen}{Z0}\cdot e^{\gamma\cdot z_n}$$





Note how the standing wave changes as you go away from the load on the lossy TL. Max & min occur at λ/2 intervals.

Matched load, Lossless TL ($\alpha = 0$)

$$\Gamma Lm := \frac{Zm - Z0}{Zm + Z0} \qquad \boxed{|\Gamma Lm| = 0}$$

$$Sm := \frac{1 + |\Gamma Lm|}{1 - |\Gamma Lm|} \qquad \boxed{Sm = 1}$$

No reflected/backward traveling waves means - NO standing waves!

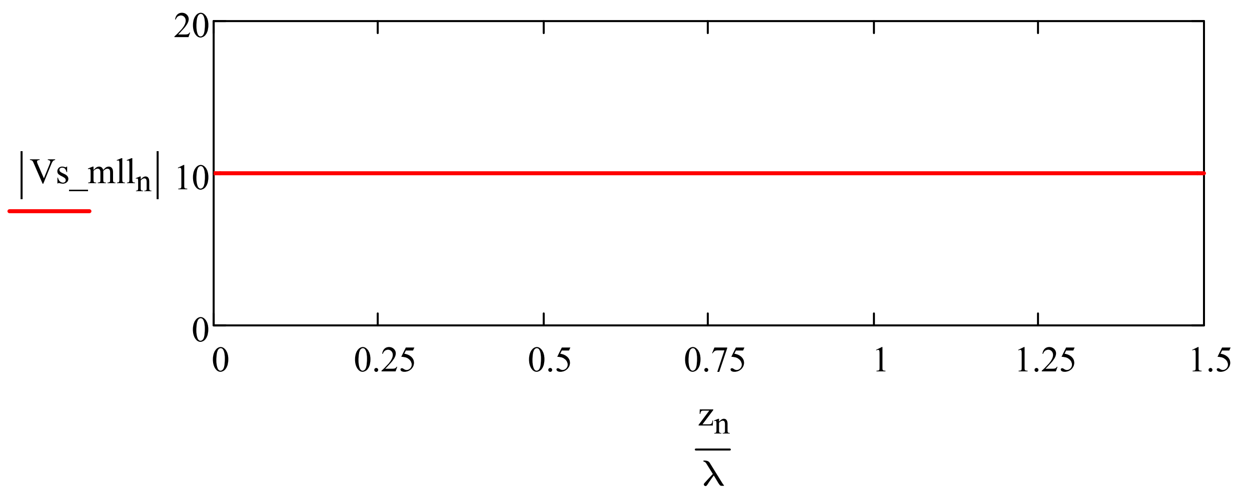
$$\Gamma inmll := \Gamma Lm \cdot e^{-j \cdot 2 \cdot \beta \cdot L} \qquad Zinmll := Z0 \cdot \frac{1 + \Gamma inmll}{1 - \Gamma inmll}$$

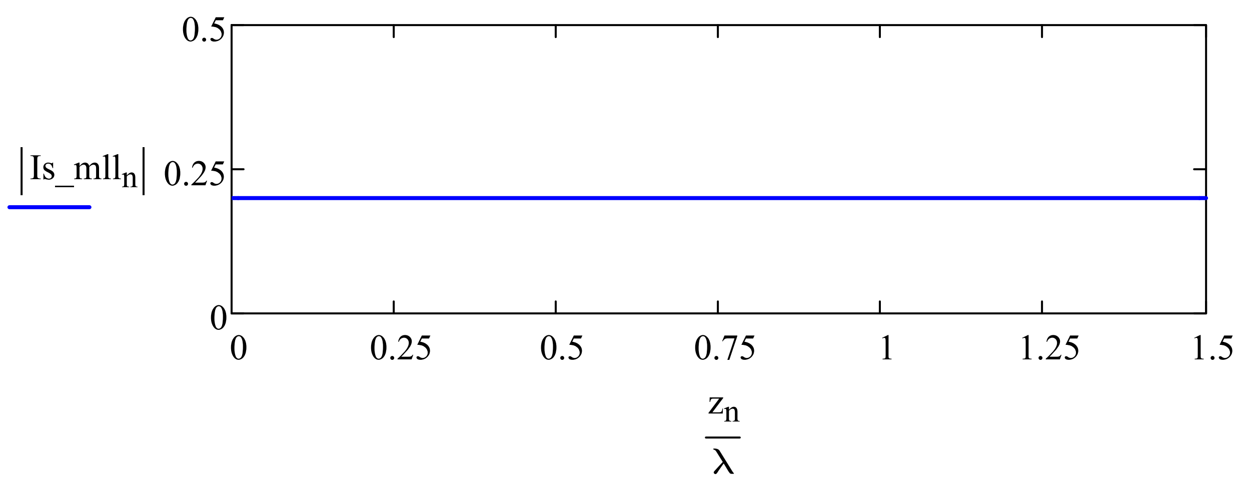
$$V0mll := Vg \cdot \frac{Zinmll}{Zg + Zinmll} \qquad I0mll := \frac{Vg}{Zg + Zinmll}$$

$$V0p\_mll := 0.5 \cdot (V0mll + Z0 \cdot I0mll) \qquad V0m\_mll := V0mll - V0p\_mll$$

$$Vs\_mll_n := V0p\_mll \cdot e^{-j \cdot \beta \cdot z_n} + V0m\_mll \cdot e^{j \cdot \beta \cdot z_n}$$

$$Is\_mll_n := \frac{V0p\_mll}{Z0} \cdot e^{-j \cdot \beta \cdot z_n} - \frac{V0m\_mll}{Z0} \cdot e^{j \cdot \beta \cdot z_n}$$





Matched load, Lossy TL

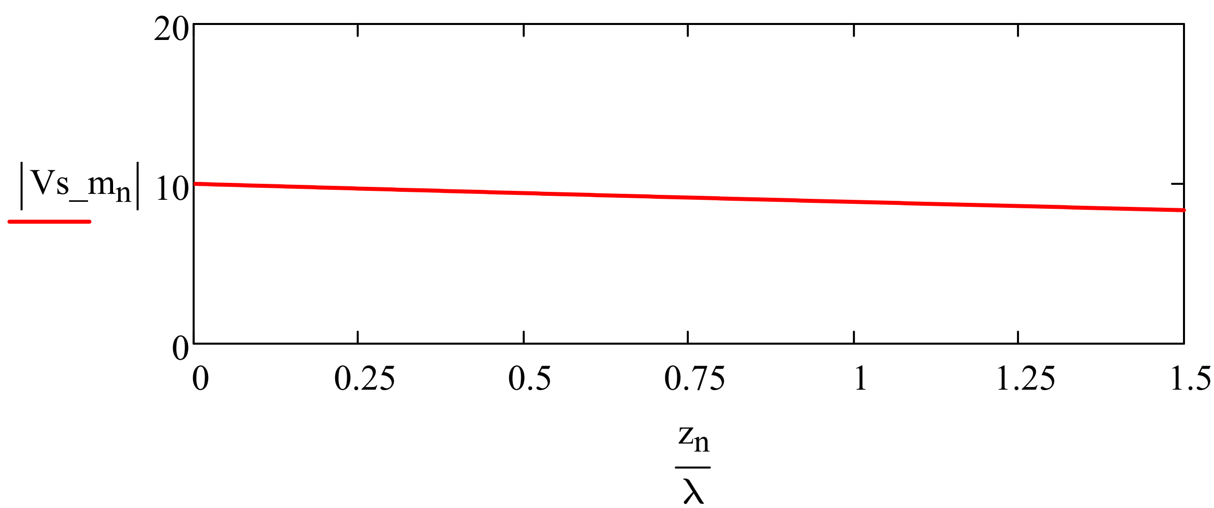
$$\Gamma inm := \Gamma Lm \cdot e^{-2\cdot\gamma\cdot L} \qquad Zinm := Z0\cdot\frac{1+\Gamma inm}{1-\Gamma inm}$$

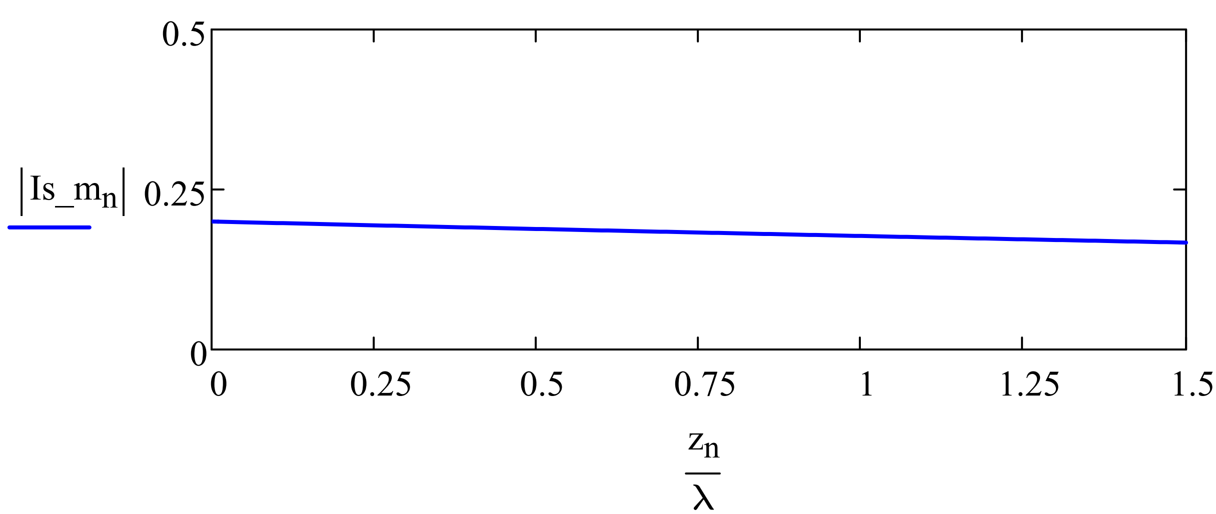
$$V0m := Vg\cdot\frac{Zinm}{Zg + Zinm} \qquad I0m := \frac{Vg}{Zg + Zinm}$$

$$V0p\_m := 0.5\cdot(V0m + Z0\cdot I0m) \qquad V0m\_m := V0m - V0p\_m$$

$$Vs\_m_n := V0p\_m\cdot e^{-\gamma\cdot z_n} + V0m\_m\cdot e^{\gamma\cdot z_n}$$

$$Is\_m_n := \frac{V0p\_m}{Z0}\cdot e^{-\gamma\cdot z_n} - \frac{V0m\_m}{Z0}\cdot e^{\gamma\cdot z_n}$$





NO standing waves! However, note the forward propagating waves decay as you go from the source toward the load on lossy TL.