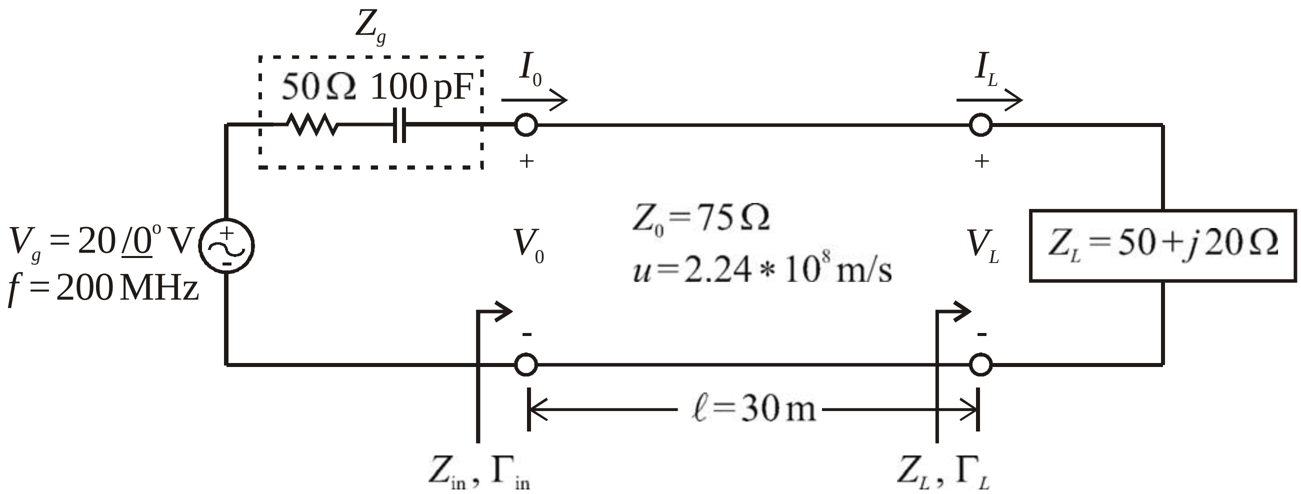
EE381 Lossless Transmission Line example with power

For the circuit shown below, calculate the standing wave ratio as well as input & load currents, voltages, powers, reflection coefficients, and impedances. Also, plot the magnitude of the current and voltage as a function of position.



Enter given information

$Vg := 20 \cdot e^{j \cdot 0 \cdot \frac{\pi}{180}}$ V $f := 200 \cdot 10^6$ Hz $L := 30$ m $u := 2.24 \cdot 10^8$ m/s

$Rg := 50$ Ω $Cg := 100 \cdot 10^{-12}$ F $Z0 := 75$ Ω $ZL := 50 + j \cdot 20$ Ω

Calculate variables related to generator and transmission line

$\omega := 2 \cdot \pi \cdot f$ $\omega = 1.257 \times 10^9$ rad/s $\beta := \frac{\omega}{u}$ $\beta = 5.61$ rad/m

$Zg := Rg - \frac{j}{2 \cdot \pi \cdot f \cdot Cg}$ $Zg = 50 - 7.9577i$ Ω $\lambda := \frac{u}{f}$ $\lambda = 1.12$ m

Ex. cont.

Calculate reflection coefficients and SWR

$\Gamma L := \dfrac{ZL - Z0}{ZL + Z0}$ $\quad \boxed{\left|\Gamma L\right| = 0.2529} \quad \boxed{\arg(\Gamma L) \cdot \dfrac{180}{\pi} = 132.25}$ deg

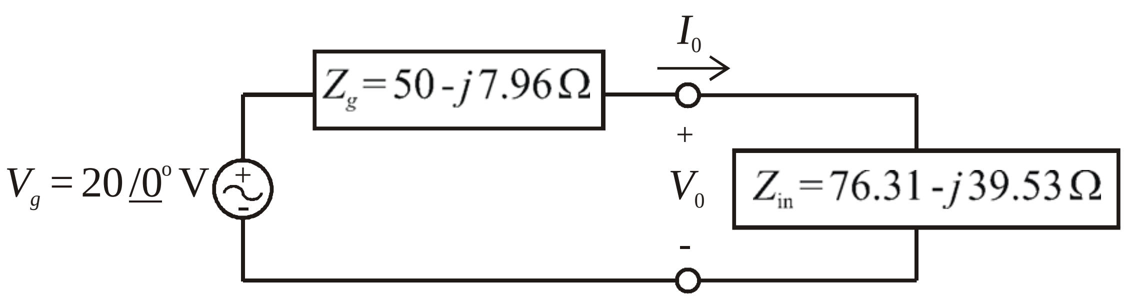
$\Gamma 0 := \Gamma L \cdot e^{-j \cdot 2 \cdot \beta \cdot L}$ $\quad \boxed{\left|\Gamma 0\right| = 0.2529} \quad \boxed{\arg(\Gamma 0) \cdot \dfrac{180}{\pi} = -73.464}$ deg

$SWR := \dfrac{1 + \left|\Gamma L\right|}{1 - \left|\Gamma L\right|}$ $\quad \boxed{SWR = 1.677}$

Calculate input impedance

$Zin := Z0 \cdot \dfrac{(1 + \Gamma 0)}{1 - \Gamma 0}$ $\quad \boxed{Zin = 76.3073 - 39.5296i}$ Ω

Calculate input current, voltage, and power using equivalent circuit



$I0 := \dfrac{Vg}{Zg + Zin}$ $\quad \boxed{\left|I0\right| = 0.1482}$ A $\quad \boxed{\arg(I0) \cdot \dfrac{180}{\pi} = 20.605}$ deg

$V0 := Vg \cdot \dfrac{Zin}{Zg + Zin}$ $\quad \boxed{\left|V0\right| = 12.7373}$ V $\quad \boxed{\arg(V0) \cdot \dfrac{180}{\pi} = -6.781}$ deg

$Pave\_0 := 0.5 \cdot Re\left(V0 \cdot \overline{I0}\right)$ $\quad \boxed{Pave\_0 = 0.8381}$ W

Ex. cont.

Calculate phasor forward traveling voltage wave

$Vfwd := \frac{V0}{1 + \Gamma 0}$ $\quad |Vfwd| = 11.5893$ V $\quad arg(Vfwd) \cdot \frac{180}{\pi} = 5.963$ deg

Define refl. coeff., phasor current & voltage in terms of position

$\Gamma(z) := \Gamma L \cdot e^{-j \cdot 2 \cdot \beta \cdot (L - z)}$

$Is(z) := \frac{Vfwd}{Z0} \cdot e^{-j \cdot \beta \cdot z} \cdot (1 - \Gamma(z))$ $\quad Vs(z) := Vfwd \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z))$

Calculate the load current, voltage, and power

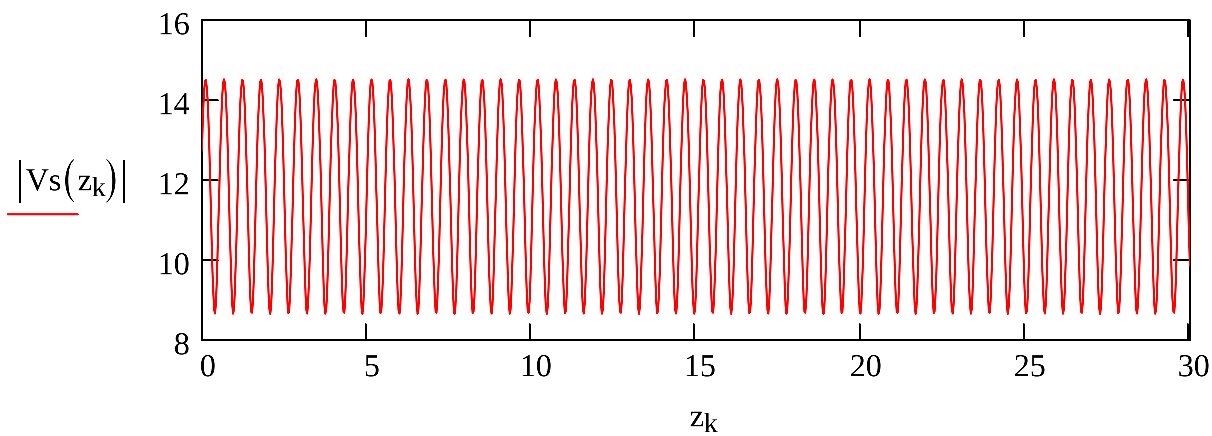
$IL := Is(L)$ $\quad \boxed{|IL| = 0.1831}$ A $\quad \boxed{arg(IL) \cdot \frac{180}{\pi} = 74.016}$ deg

$VL := Vs(L)$ $\quad \boxed{|VL| = 9.8603}$ V $\quad \boxed{arg(VL) \cdot \frac{180}{\pi} = 95.817}$ deg

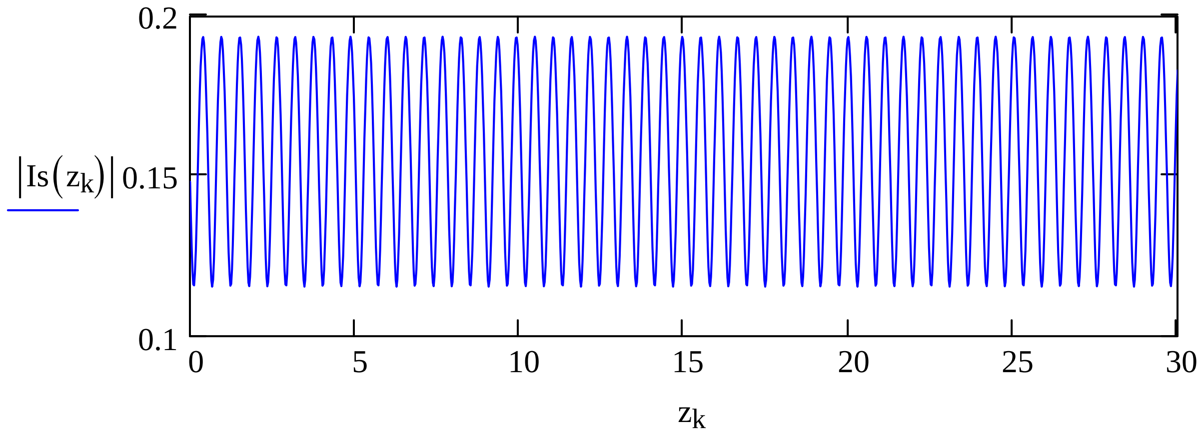
$Pave\_L := 0.5 \cdot Re(VL \cdot \overline{IL})$ $\quad \boxed{Pave\_L = 0.83815}$ W

Plot the magnitude of the current & voltage as functions of position

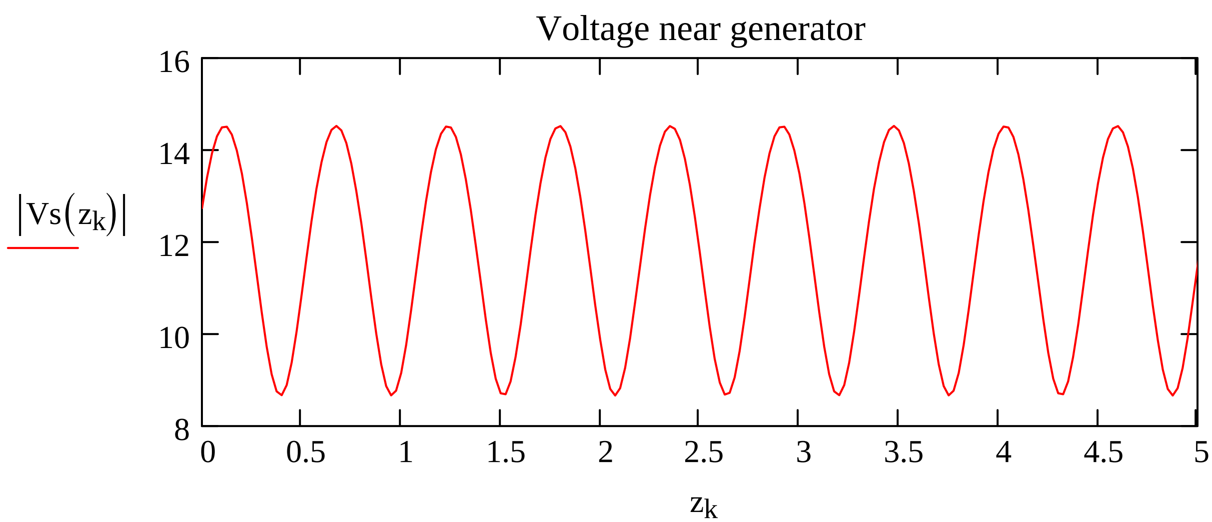
$k := 0 .. 1200$ $\quad z_k := \frac{k}{1200} \cdot L$



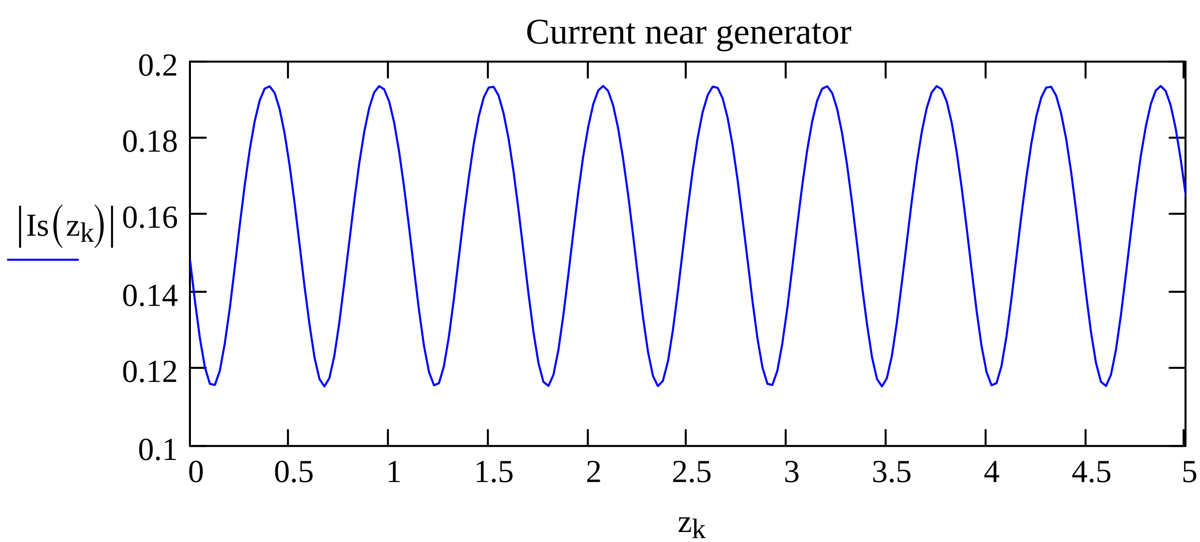
Ex. cont.



0.2
0.15
0.1
|Is(zk)|
0
5
10
15
20
25
30
zk

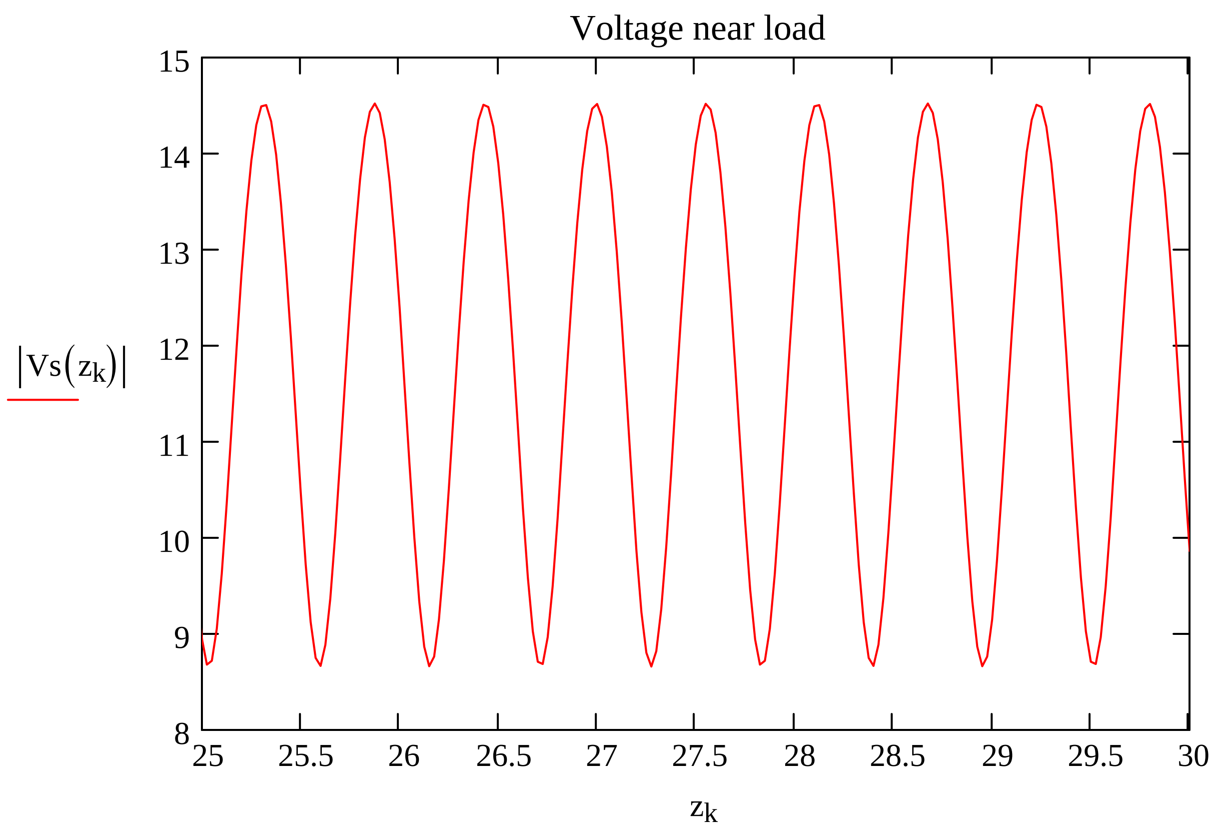


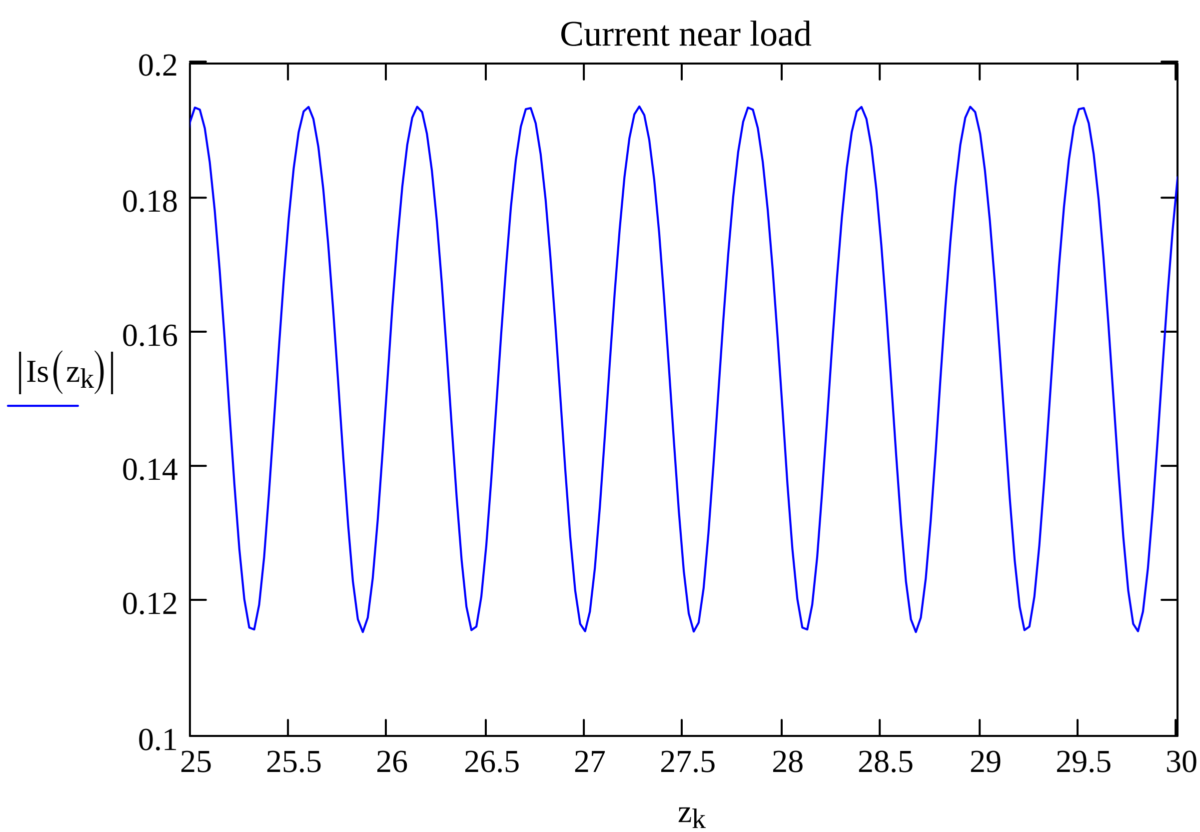
Voltage near generator
16
14
12
10
8
|Vs(zk)|
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
zk



Current near generator
0.2
0.18
0.16
0.14
0.12
0.1
|Is(zk)|
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
zk

Ex. cont.


Voltage near load
15
14
13
12
11
10
9
8
|Vs(zk)|
25
25.5
26
26.5
27
27.5
28
28.5
29
29.5
30
zk

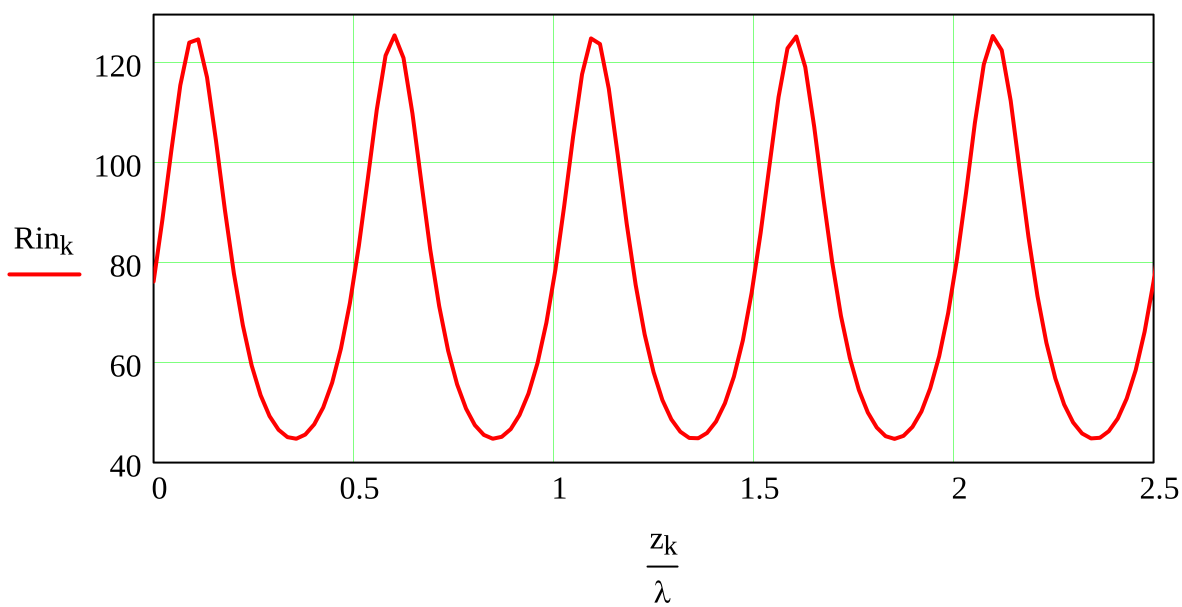

Current near load
0.2
0.18
0.16
0.14
0.12
0.1
|Is(zk)|
25
25.5
26
26.5
27
27.5
28
28.5
29
29.5
30
zk

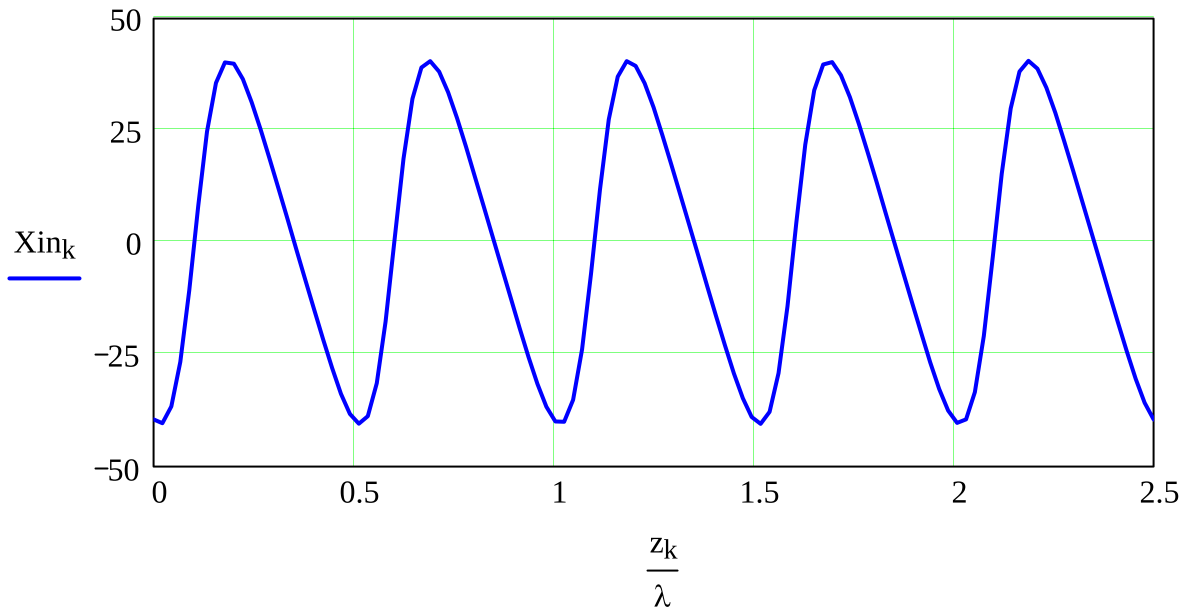
Ex. cont.

Plot the input impedance as a function of position near the generator

$$Zin_k := Z0 \cdot \left(\frac{1 + \Gamma(z_k)}{1 - \Gamma(z_k)} \right) \qquad Rin_k := \mathrm{Re}(Zin_k) \qquad Xin_k := \mathrm{Im}(Zin_k)$$

Remember Zin is complex, separate the real & imaginary parts for plotting.





Everything repeats at λ/2 intervals!