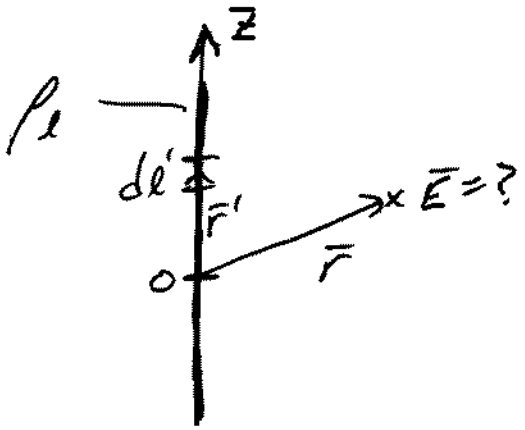
Example- Find the electric field due to a uniform line charge ρ_l of infinite length situated on the z-axis in free space.

Use $\bar{E} = \int_L \frac{\rho_L(\bar{r} - \bar{r}')\,dl'}{4\pi\varepsilon_0 |\bar{r} - \bar{r}'|^3}$.



$dl' = dz'$

$\bar{r} = \rho\hat{a}_\rho + z\hat{a}_z$

$\bar{r}' = \rho'\hat{a}_{\rho'} + z'\hat{a}_z$ ($\rho' = 0$)

$\bar{r} - \bar{r}' = \rho\hat{a}_\rho + (z - z')\hat{a}_z$

$$\bar{E} = \int_{z'=-\infty}^{\infty} \frac{\rho_l}{4\pi\epsilon_0} \frac{(\rho\hat{a}_\rho + (z-z')\hat{a}_z)\,dz'}{[\rho^2 + (z-z')^2]^{3/2}}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \int_{z'=-\infty}^{\infty} \frac{\rho\hat{a}_\rho + (z-z')\hat{a}_z\,dz'}{[\rho^2 + (z-z')^2]^{3/2}}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \left\{ \frac{(z'-z)\rho\hat{a}_\rho}{[\rho^2]\sqrt{\rho^2 + (z-z')^2}} \Bigg|_{z'=-\infty}^{\infty} + \frac{\hat{a}_z}{\sqrt{\rho^2 + (z-z')^2}} \Bigg|_{-\infty}^{\infty} \right\}$$

$\to 2\hat{a}_\rho/\rho$; $\to 0$

$$\bar{E} = \hat{a}_\rho \frac{\rho_l}{2\pi\epsilon_0\rho}$$