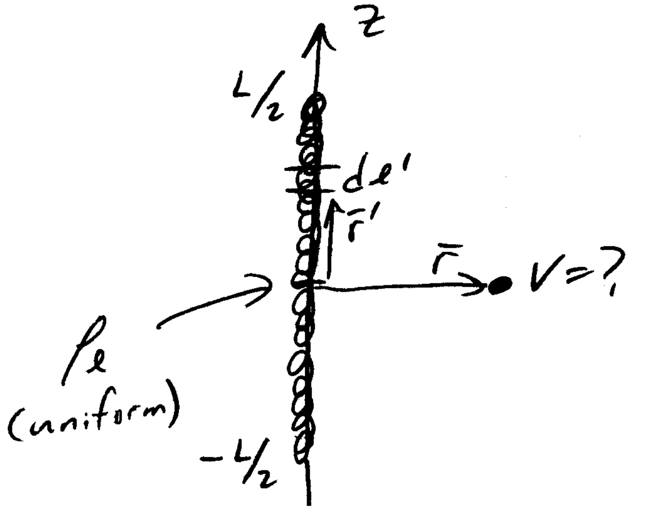
Example- Find the electric potential (voltage) due to a uniform line charge ρ_l of finite length L, centered about the origin on the z-axis in free space, at a point on the x-y plane.



$$V = \frac{1}{4\pi\epsilon_0}\int_L \frac{\rho_L(\bar{r}')\,dl'}{|\bar{r}-\bar{r}'|}$$

$$\rho_L(\bar{r}') = \rho_l , \quad dl' = dz'$$

$$\bar{r} = \rho\hat{a}_\rho + \overset{0}{\cancel{z}}\hat{a}_z = \rho\hat{a}_\rho$$

$$\bar{r}' = \overset{0}{\cancel{\rho'}}\hat{a}_\rho' + z'\hat{a}_z = z'\hat{a}_z$$

$$V = \frac{1}{4\pi\epsilon_0}\int_{z'=-L/2}^{L/2} \frac{\rho_l\,dz'}{|\rho\hat{a}_\rho - z'\hat{a}_z|}$$

$$= \frac{\rho_l}{4\pi\epsilon_0}\int_{z'=-L/2}^{L/2} \frac{dz'}{(\rho^2+z'^2)^{1/2}}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \ln\left(z' + \sqrt{z'^2+\rho^2}\right)\Big|_{-L/2}^{L/2}$$

$$= \frac{\rho_l}{4\pi\epsilon_0}\left[\ln\left(L/2 + \sqrt{(L/2)^2+\rho^2}\right) - \ln\left(-L/2 + \sqrt{(-L/2)^2+\rho^2}\right)\right]$$

$$\underline{\underline{V = \frac{\rho_l}{4\pi\epsilon_0}\ln\left[\frac{L/2 + \sqrt{(L/2)^2+\rho^2}}{-L/2 + \sqrt{(L/2)^2+\rho^2}}\right] \qquad \rho > 0}}$$

In ln[] argument, divide numerator and denominator by $L/2$. Normalize by $\rho_\ell/4\pi\varepsilon_0$.

$n := 1 .. 100 \qquad \rho L_n := 0.05 \cdot n \qquad \mathrm{Vnorm}(x) := \ln\left[\dfrac{1+\sqrt{1+(2\cdot x)^2}}{-1+\sqrt{1+(2\cdot x)^2}}\right]$