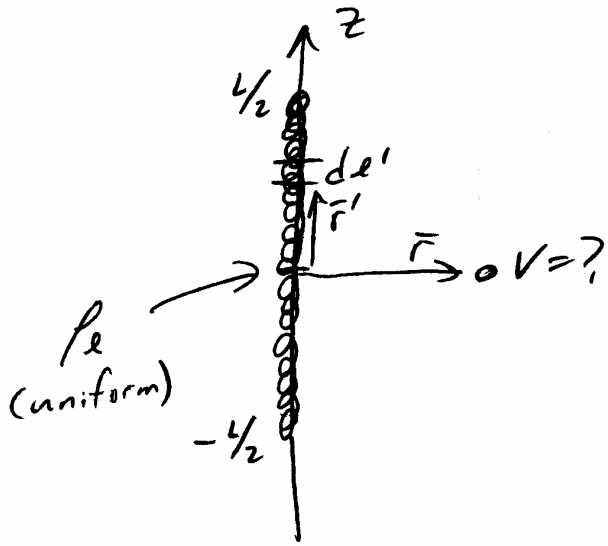


Example- Find the electric potential (voltage) due to a uniform line charge ρ_l of finite length L , centered about the origin on the z -axis in free space, at a point on the x - y plane.



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_l(\vec{r}') dl'}{|\vec{r} - \vec{r}'|}$$

$$\rho_l(\vec{r}') = \rho_l$$

$$dl' = dz'$$

$$\vec{r} = \rho \hat{a}_\rho$$

$$\vec{r}' = z' \hat{a}_z$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{z'=-L/2}^{L/2} \frac{\rho_l dz'}{|\rho \hat{a}_\rho - z' \hat{a}_z|}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \int_{z'=-L/2}^{L/2} \frac{dz'}{(\rho^2 + z'^2)^{1/2}}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \ln(z' + \sqrt{z'^2 + \rho^2}) \Big|_{-L/2}^{L/2}$$

$$= \frac{\rho_l}{4\pi\epsilon_0} \left[\ln\left(\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + \rho^2}\right) - \ln\left(-\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + \rho^2}\right) \right]$$

$$V(\vec{r}) = \frac{\rho_l}{4\pi\epsilon_0} \ln \left[\frac{\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + \rho^2}}{-\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + \rho^2}} \right]$$