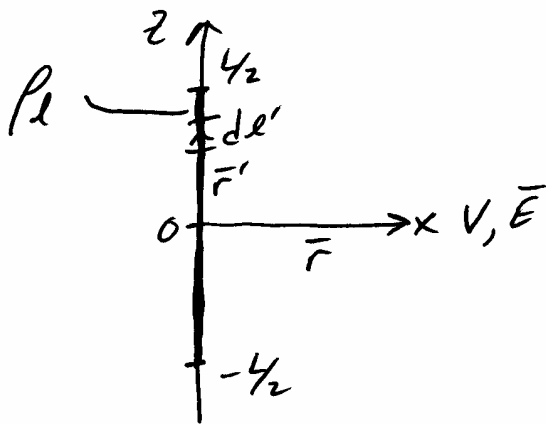


ex. Electric field due to finite length uniform line charge.



$$\begin{aligned}\vec{r}' &= z' \hat{a}_z \\ \vec{r} &= \rho \hat{a}_\rho \\ dl' &= dz' \\ \rho_l(\vec{r}') &= \rho_l\end{aligned}$$

Earlier, we found that the voltage/potential was

$$V = \frac{\rho_l}{4\pi\epsilon_0} \ln \left[ \frac{L/2 + \sqrt{(L/2)^2 + \rho^2}}{-L/2 + \sqrt{(L/2)^2 + \rho^2}} \right]$$

Using  $V$

$$\vec{E} = -\vec{\nabla}V = -\hat{a}_\rho \frac{dV}{d\rho} - \hat{a}_\phi \frac{1}{\rho} \frac{dV}{d\phi} - \hat{a}_z \frac{dV}{dz}$$

$$= -\hat{a}_\rho \frac{\rho_l}{4\pi\epsilon_0} \frac{d}{d\rho} \left\{ \ln \left[ \frac{L/2 + \sqrt{(L/2)^2 + \rho^2}}{-L/2 + \sqrt{(L/2)^2 + \rho^2}} \right] \right\}$$

$$= -\hat{a}_\rho \frac{\rho_l}{4\pi\epsilon_0} \frac{d}{d\rho} \left\{ \ln(L/2 + \sqrt{(L/2)^2 + \rho^2}) - \ln(-L/2 + \sqrt{(L/2)^2 + \rho^2}) \right\}$$

$$= -\hat{a}_\rho \frac{\rho_l}{4\pi\epsilon_0} \left[ \frac{1}{L/2 + \sqrt{(L/2)^2 + \rho^2}} (L/2) [(L/2)^2 + \rho^2]^{-1/2} (2\rho) - \frac{1}{-L/2 + \sqrt{(L/2)^2 + \rho^2}} (L/2) [(L/2)^2 + \rho^2]^{-1/2} (2\rho) \right]$$

ex. cont.

$$\begin{aligned}\bar{E} &= -\hat{a}_\rho \frac{\rho_L}{4\pi\epsilon_0} \frac{\rho}{\sqrt{(y_2)^2 + \rho^2}} \left[ \frac{(-y_2 + \sqrt{(y_2)^2 + \rho^2}) - (y_2 + \sqrt{(y_2)^2 + \rho^2})}{(y_2 + \sqrt{(y_2)^2 + \rho^2})(-y_2 + \sqrt{(y_2)^2 + \rho^2})} \right] \\ &= -\hat{a}_\rho \frac{\rho_L}{4\pi\epsilon_0} \frac{\rho}{\sqrt{(y_2)^2 + \rho^2}} \left[ \frac{-L}{-(y_2)^2 + 0 + (y_2)^2 + \rho^2} \right]\end{aligned}$$

$$\bar{E} = +\hat{a}_\rho \frac{\rho_L}{4\pi\epsilon_0} \frac{L}{\rho\sqrt{(y_2)^2 + \rho^2}} \quad (\text{V/m})$$


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Calculating  $\bar{E}$  directly gives:

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_L(\bar{r}')(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} dl'$$

$$= \frac{1}{4\pi\epsilon_0} \int_{z'=-y_2}^{y_2} \frac{\rho_L(\rho\hat{a}_\rho - z'\hat{a}_z)}{|\rho\hat{a}_\rho - z'\hat{a}_z|^3} dz' \quad \leftarrow \text{No primed unit vectors!}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left[ \rho\hat{a}_\rho \int_{z'=-y_2}^{y_2} \frac{dz'}{[\rho^2 + z'^2]^{3/2}} - \hat{a}_z \int_{z'=-y_2}^{y_2} \frac{z' dz'}{[\rho^2 + z'^2]^{3/2}} \right]$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left[ \rho\hat{a}_\rho \frac{z'}{\rho^2\sqrt{\rho^2 + z'^2}} \Big|_{z'=-y_2}^{y_2} + \hat{a}_z \left( \frac{1}{\sqrt{\rho^2 + z'^2}} \right) \Big|_{z'=-y_2}^{y_2} \right]$$

$$\bar{E} = \hat{a}_\rho \frac{\rho_L}{4\pi\epsilon_0} \frac{L}{\rho\sqrt{\rho^2 + (y_2)^2}} \quad \therefore \leftarrow \text{Same answer!}$$


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