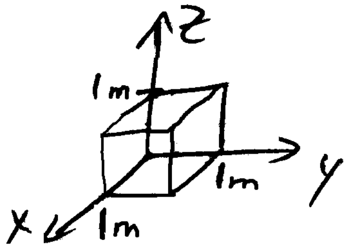
divergence_theorem_example.doc

Verify Divergence Theorem, $\oint\!\!\oint_S \overline{A} \bullet d\overline{s} = \iiint_v \overline{\nabla} \bullet \overline{A}\, dv$, for $\overline{A} = x^2 y\, \hat{a}_x$ (cats/m^2) for the surface S surrounding the volume $0 < x < 1$ m, $0 < y < 1$ m, and $0 < z < 1$ m.



$$\overline{\nabla} \bullet \overline{A} = \frac{\partial x^2 y}{\partial x} = \underline{\underline{2xy \left(\frac{\text{cats}}{m^3}\right)}}$$

$$\iiint_V \overline{\nabla} \bullet \overline{A}\, dv = \int_{z=0}^{1} \int_{y=0}^{1} \int_{x=0}^{1} 2xy\, dx\, dy\, dz$$

$$= 2 \int_{z=0}^{1} dz \int_{y=0}^{1} y\, dy \int_{x=0}^{1} x\, dx$$

$$= 2\,(1-0)\left(\frac{y^2}{2}\right)\Big|_0^1 \left(\frac{x^2}{2}\right)\Big|_0^1 = 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$

$$\iiint_V \overline{\nabla} \bullet \overline{A}\, dv = \underline{\underline{0.5 \text{ (cats)}}}$$

$\hat{a}_x \bullet \pm\hat{a}_z = 0$

$\hat{a}_x \bullet \pm\hat{a}_y = 0$

$$\oint\!\!\oint_S \overline{A} \bullet d\overline{s} = \underbrace{\left[\iint_{Top} \overline{A} \bullet d\overline{s}_z + \iint_{Bott} \overline{A} \bullet -d\overline{s}_z + \iint_{Right} \overline{A} \bullet d\overline{s}_y + \iint_{Left} \overline{A} \bullet -d\overline{s}_y \right]}_{0}$$

$$+ \iint_{front} \overline{A} \bullet d\overline{s}_x + \iint_{Back} \overline{A} \bullet -d\overline{s}_x \qquad d\overline{s}_x = \hat{a}_x\, dy\, dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1} \underbrace{x^2}_{\substack{\to 1^2 \\ (x=1)}} y\, dy\, dz - \int_{z=0}^{1}\int_{y=0}^{1} \underbrace{x^2}_{\substack{\to 0 \\ (x=0)}} y\, dy\, dz = \underline{\underline{0.5 \text{ (cats)}}}$$