Example- Convert vector $\bar{A}_{\text{Cart}} = 3\hat{a}_y - 4\hat{a}_z$ to cylindrical coordinates, i.e., find $\bar{A}_{\text{cyl}}$. Also, evaluate the magnitude of $\bar{A}_{\text{cyl}}$ at the point (2, 120°, 5) and compare to the magnitude of $\bar{A}_{\text{Cart}}$.

Method 1 Convert unit vectors from Cartesian → Cylind.

$$\bar{A}_{cyl} = 3\left(\sin\phi\, \hat{a}_\rho + \cos\phi\, \hat{a}_\phi\right) - 4\,\hat{a}_z$$

$$\underline{\underline{\bar{A}_{cyl} = 3\sin\phi\, \hat{a}_\rho + 3\cos\phi\, \hat{a}_\phi - 4\,\hat{a}_z}}$$

$$\bar{A}_{cyl}(2, 120°, 5) = 3\sin 120°\, \hat{a}_\rho + 3\cos 120°\, \hat{a}_\phi - 4\,\hat{a}_z$$

$$\underline{\bar{A}_{cyl}(2, 120°, 5) = 2.598\, \hat{a}_\rho - 1.5\, \hat{a}_\phi - 4\,\hat{a}_z}$$

$$\left|\bar{A}_{cyl}(2, 120°, 5)\right| = \sqrt{2.598^2 + (-1.5)^2 + (-4)^2} = \underline{\underline{5}}$$

$$\left|\bar{A}_{cart}\right| = \sqrt{3^2 + (-4)^2} = \underline{\underline{5}}$$

Method 2 Calculate cylindrical vector components

$$A_\rho = A_x \cos\phi + A_y \sin\phi = 0\cos\phi + 3\sin\phi = 3\sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi = -0\sin\phi + 3\cos\phi = 3\cos\phi$$

$$A_z = A_z = -4$$

$$\bar{A}_{cyl} = A_\rho\, \hat{a}_\rho + A_\phi\, \hat{a}_\phi + A_z\, \hat{a}_z = \underline{\underline{3\sin\phi\, \hat{a}_\rho + 3\cos\phi\, \hat{a}_\phi - 4\,\hat{a}_z}}$$

⇑ Same!