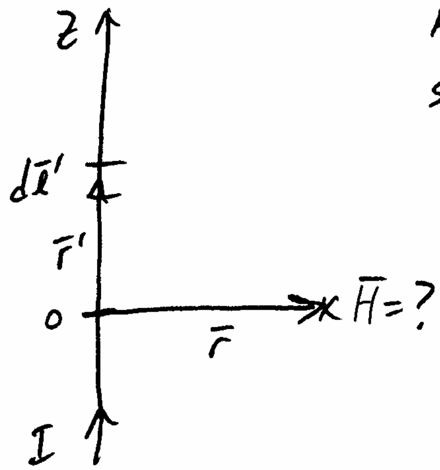
ex. Find magnetic field vector due to current I flowing along z-axis adjacent to origin.



First, look at a differential section using cylindrical coord.

$$d\bar{H} = \frac{I\, d\bar{l}' \times (\bar{r}-\bar{r}')}{4\pi |\bar{r}-\bar{r}'|^3}$$

$$d\bar{l}' = \hat{a}_\rho' \underbrace{\cancel{d\rho'}}_{0} + \underbrace{\cancel{\rho'}}_{0} d\phi' \, \hat{a}_\phi' + dz' \hat{a}_z = dz' \hat{a}_z$$

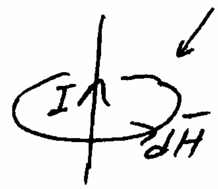
$$\bar{r} = \rho \hat{a}_\rho + \underbrace{\cancel{z}}_{0} \hat{a}_z = \rho \hat{a}_\rho$$

$$\bar{r}' = \underbrace{\cancel{\rho'}}_{0} \hat{a}_{\rho'} + z' \hat{a}_z = z' \hat{a}_z$$

$$d\bar{H} = \frac{I\, dz' \hat{a}_z \times (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi |\rho \hat{a}_\rho - z' \hat{a}_z|^3}$$

$$= \frac{I\, dz' \rho \hat{a}_\phi - 0}{4\pi \left(\sqrt{\rho^2 + (-z')^2}\right)^3}$$

$$d\bar{H} = \hat{a}_\phi \frac{I}{4\pi} \frac{\rho \, dz'}{(\rho^2 + z'^2)^{3/2}}$$



$\bar{H}$ forms closed loops around current element (IEEE Symbol)

Next, find $\bar{H}$ due a length L (centered on the origin) of this current I

$$\bar{H} = \int_{C'} \frac{I\, d\bar{l}' \times (\bar{r}-\bar{r}')}{4\pi |\bar{r}-\bar{r}'|^3} = \hat{a}_\phi \frac{I\rho}{4\pi} \int_{z'=-L/2}^{L/2} \frac{dz'}{(\rho^2+z'^2)^{3/2}}$$

ex. cont.

$$= \hat{a}_\phi \frac{I\rho}{4\pi} \left. \frac{z'}{\rho^2\sqrt{\rho^2+z'^2}} \right|_{z'=-L/2}^{L/2}$$

$$= \hat{a}_\phi \frac{I}{4\pi\rho} \left[\frac{L/2}{\sqrt{\rho^2+(L/2)^2}} - \frac{-L/2}{\sqrt{\rho^2+(-L/2)^2}} \right]$$

$$\underline{\underline{\bar{H} = \hat{a}_\phi \frac{I}{2\pi\rho} \frac{L/2}{\sqrt{\rho^2+L^2/4}}}} \quad @ \ z=0$$

Last, what if we assume the wire is infinitely long?

$$\bar{H} = \oint_{c'} \frac{I d\bar{\ell}' \times (\bar{r}-\bar{r}')}{4\pi|\bar{r}-\bar{r}'|^3} = \hat{a}_\phi \frac{I\rho}{4\pi} \int_{z'=-\infty}^{\infty} \frac{dz'}{(\rho^2+z'^2)^{3/2}}$$

$$= \hat{a}_\phi \frac{I}{4\pi\rho} \left. \frac{z'}{\sqrt{\rho^2+z'^2}} \right|_{z'=-\infty}^{\infty}$$

$$= \hat{a}_\phi \frac{I}{4\pi\rho} \left[\frac{\infty}{\infty} - \underbrace{\frac{-\infty}{\infty}}_{\to 2} \right]$$

$$\underline{\underline{\bar{H} = \hat{a}_\phi \frac{I}{2\pi\rho}}}$$ ← very important for visualizing problems