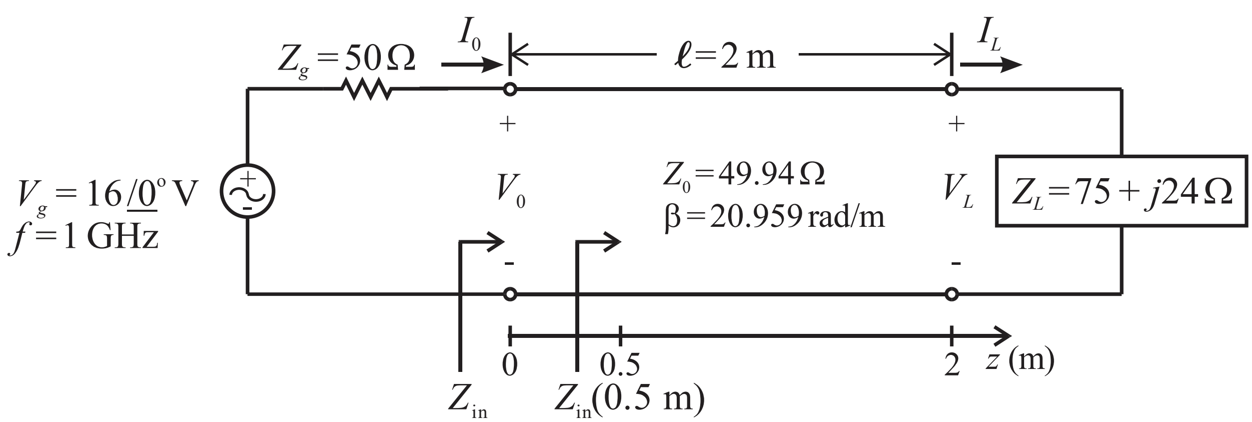
Use the **lossless** air-dielectric coaxial transmission line (TL) operating at 1 GHz [made with perfect electrical conductors, an inner conductor of radius 1 cm, and shield of radius 2.3 cm] from the earlier example in the TL circuit shown below to find the input impedance at z = 0 and 0.5 m, electrical length $\beta\ell$, phasor input voltage V_0 and current I_0, amplitudes of forward and backward waves (V_0^+, V_0^-, I_0^+, and I_0^-), expressions for phasor voltage $V_s(z)$ and current $I_s(z)$, and phasor load voltage V_L and current I_L.

****

Prior example- $\beta = 20.959$ rad/m, $u = 2.998\times10^8$ m/s, and $Z_0 = 49.94\ \Omega$.

****



Find input impedances using (11.34) and equation given in notes-

$$Z_{\text{in}} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}\right] = 49.94\left[\frac{(75+j24)+j49.94\tan\left(20.959(2)\right)}{49.94+j(75+j24)\tan\left(20.959(2)\right)}\right]$$

$$\Rightarrow \underline{\underline{Z_{\text{in}} = 42.81 - j25.23\ \Omega}}$$

and

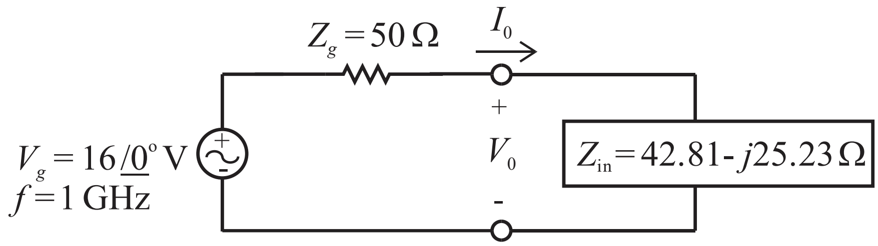
$$Z_{\text{in}}(z=0.5\,\text{m}) = Z_0\left[\frac{Z_L + jZ_0\tan\left[\beta(\ell - z)\right]}{Z_0 + jZ_L\tan\left[\beta(\ell - z)\right]}\right]$$

$$= 49.94\left[\frac{(75+j24)+j49.94\tan\left[20.959(2-0.5)\right]}{49.94+j(75+j24)\tan\left[20.959(2-0.5)\right]}\right].$$

$$\Rightarrow \underline{\underline{Z_{\text{in}}(0.5\,\text{m}) = 76.6 + j22.78\ \Omega}}$$

The electrical length is $\beta\ell = 20.959(2)$ $\Rightarrow$ $\underline{\underline{\beta\ell = 41.918\text{ rad} = 2401.72^\circ}}$.

Next, using the input impedance Z_{in} that was just calculated, we can draw and equivalent circuit for the TL input.



Using circuit analysis, calculate the phasor input voltage V_0 and current I_0-

$$V_0 = V_g \left[\frac{Z_{\text{in}}}{Z_g + Z_{\text{in}}}\right] = (16\angle 0^\circ)\left[\frac{(42.81 - j25.23)}{50 + (42.81 - j25.23)}\right] \quad \Rightarrow \quad \underline{\underline{V_0 = 8.267\angle\text{-}15.3^\circ \text{ V}}}$$

$$I_0 = \frac{V_g}{Z_g + Z_{\text{in}}} = \frac{16\angle 0^\circ}{50 + (42.81 - j25.23)} \quad \Rightarrow \quad \underline{\underline{I_0 = 0.1664\angle 15.21^\circ \text{ A}}}$$

Find amplitudes of forward and backward waves (V_0^+, V_0^-, I_0^+, and I_0^-) using (11.27a), (11.27b), and equations given in notes-

$$V_0^+ = 0.5\left[V_0 + Z_0 I_0\right] = 0.5\left[(8.267\angle\text{-}15.3^\circ) + 49.94(0.1664\angle 15.21^\circ)\right]$$

$$\Rightarrow \quad \underline{\underline{V_0^+ = 7.995\angle\text{-}0.009^\circ \text{ V}}},$$

$$V_0^- = 0.5\left[V_0 - Z_0 I_0\right] = 0.5\left[(8.267\angle\text{-}15.3^\circ) - 49.94(0.1664\angle 15.21^\circ)\right]$$

$$\Rightarrow \quad \underline{\underline{V_0^- = 2.181\angle\text{-}90.57^\circ \text{ V}}},$$

$$I_0^+ = V_0^+ / Z_0 = (7.995\angle\text{-}0.009^\circ) / 49.94 \quad \Rightarrow \quad \underline{\underline{I_0^+ = 0.1601\angle\text{-}0.009^\circ \text{ A}}},$$

$$I_0^- = -V_0^- / Z_0 = -(2.181\angle\text{-}90.57^\circ) / 49.94 \quad \Rightarrow \quad \underline{\underline{I_0^- = 0.04366\angle 89.43^\circ \text{ A}}}.$$

The expressions for the phasor voltage $V_s(z)$ and current $I_s(z)$ are

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$\underline{\underline{V_s(z) = (7.995\angle\text{-}0.009^\circ)e^{-j20.959z} + (2.181\angle\text{-}90.57^\circ)e^{+j20.959z} \text{ V for } 0 \le z \le 2\,\text{m}}}$$

and

$$I_s(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z}$$

$$\underline{\underline{I_s(z) = (0.1601\angle\text{-}0.009^\circ)e^{-j20.959z} + (0.0437\angle 89.43^\circ)e^{+j20.959z} \text{ A for } 0 \le z \le 2\,\text{m}}}.$$

The phasor load voltage V_L and current I_L are

$$V_L = V_s(z=\ell=2\,\text{m}) = (7.995\angle\text{-}0.009°)e^{-j20.959(2)} + (2.181\angle\text{-}90.57°)e^{+j20.959(2)}$$

$$\Rightarrow \underline{\underline{V_L = 9.897\angle 125.14°\ \text{V}}}$$

and

$$I_L = I_s(z=\ell=2\,\text{m}) = (0.1601\angle\text{-}0.009°)e^{-j20.959(2)} + (0.0437\angle 89.43°)e^{+j20.959(2)}$$

$$\Rightarrow \underline{\underline{I_L = 0.1257\angle 107.39°\ \text{A}}}\ .$$

These answers were found on my calculator as well as checked using MathCAD. Try working them **on your calculator** to ensure you understand how it works (i.e., appropriate entry formats, modes, and settings).