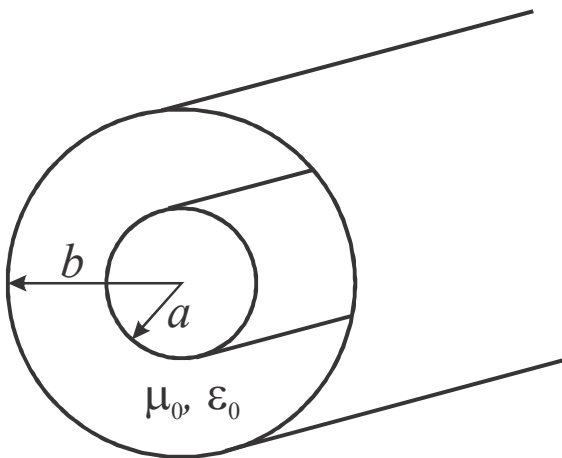


Determine the per-unit-length parameters  $R$ ,  $L$ ,  $C$ , and  $G$  for an air-dielectric coaxial transmission line operating at 1 GHz made with perfect electrical conductors when the inner conductor has a radius of 1 cm and the shield has a radius of 2.3 cm. Then, find the phase constant  $\beta$ , wavelength  $\lambda$ , phase velocity  $u$ , and characteristic impedance  $Z_0$ .



$$a = 1 \text{ cm}$$

$$b = 2.3 \text{ cm}$$

$$\sigma_c = \sigma_{\text{inner}} = \sigma_{\text{shield}} \rightarrow \infty$$

$$\sigma = \sigma_{\text{air}} = 0$$

$$\mu = \mu_{\text{air}} = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\epsilon = \epsilon_{\text{air}} = \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

### From Table 11.1

$$R = \frac{1}{2\pi \delta \sigma_c} \left[ \frac{1}{a} + \frac{1}{b} \right] = \frac{\sqrt{\pi f \mu_c}}{2\pi \sqrt{\sigma_c}} \left[ \frac{1}{a} + \frac{1}{b} \right] \Bigg|_{\sigma_c \rightarrow \infty} \Rightarrow \underline{\underline{R = 0}}$$

$$L = \frac{\mu}{2\pi} \ln(b/a) = \frac{4\pi \times 10^{-7}}{2\pi} \ln(2.3/1) = 1.666 \times 10^{-7} \text{ H/m} \Rightarrow \underline{\underline{L = 166.6 \text{ nH/m}}}$$

$$G = \frac{2\pi \sigma}{\ln(b/a)} = \frac{2\pi \sigma_{\text{air}}}{\ln(b/a)} = \frac{2\pi (0)}{\ln(2.3/1)} \Rightarrow \underline{\underline{G = 0}}$$

$$C = \frac{2\pi \epsilon}{\ln(b/a)} = \frac{2\pi (8.854 \times 10^{-12})}{\ln(2.3/1)} = 6.679 \times 10^{-11} \text{ F/m} \Rightarrow \underline{\underline{C = 66.79 \text{ pF/m}}}$$

$$= \frac{\mu \epsilon}{L} = \frac{4\pi \times 10^{-7} (8.854 \times 10^{-12})}{1.666 \times 10^{-7}} = 6.679 \times 10^{-11} \text{ F/m (same)}$$

\*\*\*\*\*

$$\beta = \omega \sqrt{LC} = 2\pi (1 \times 10^9) \sqrt{1.666 \times 10^{-7} (6.679 \times 10^{-11})} \Rightarrow \underline{\underline{\beta = 20.959 \text{ rad/m}}}$$

$$u = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.666 \times 10^{-7} (6.679 \times 10^{-11})}} \Rightarrow \underline{\underline{u = 2.998 \times 10^8 \text{ m/s}}}$$

$$\lambda = 2\pi/\beta = u/f = 2.998 \times 10^8 / 1 \times 10^9 \Rightarrow \underline{\underline{\lambda = 0.2998 \text{ m}}}$$

$$Z_0 = \sqrt{L/C} = \sqrt{1.666 \times 10^{-7} / 6.679 \times 10^{-11}} \Rightarrow \underline{\underline{Z_0 = 49.94 \Omega}}$$