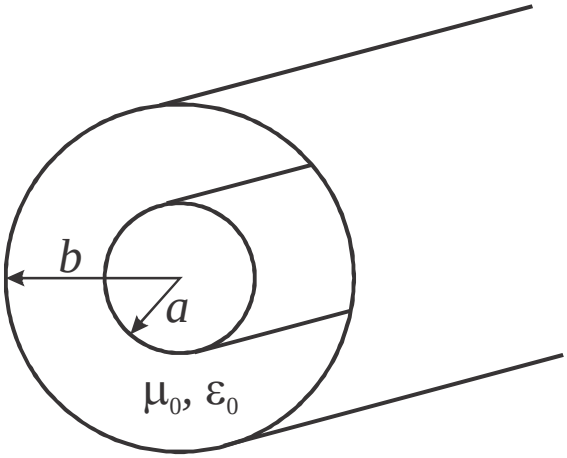
Determine the per-unit-length parameters *R*, *L*, *C*, and *G* for an air-dielectric coaxial transmission line operating a 1 GHz made with perfect electrical conductors when the inner conductor has a radius of 1 cm and the shield has a radius of 2.3 cm. Then, find the phase constant β, wavelength λ, phase velocity *u*, and characteristic impedance Z_0.



a = 1 cm

b = 2.3 cm

$\sigma_c = \sigma_{\text{inner}} = \sigma_{\text{shield}} \to \infty$

$\sigma_{\text{air}} = 0$

$\mu_{\text{air}} = \mu_0 = 4\pi \times 10^{-7}$ H/m

$\varepsilon_{\text{air}} = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m

From Table 11.1

$$R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right] = \frac{\sqrt{\pi f \mu_c}}{2\pi\sqrt{\sigma_c}}\left[\frac{1}{a}+\frac{1}{b}\right]\Bigg|_{\sigma_c\to\infty} \quad \Rightarrow \quad \underline{\underline{R=0}}$$

$$L = \frac{\mu}{2\pi}\ln\ b/a\ = \frac{4\pi\times10^{-7}}{2\pi}\ln\ 2.3/1\ = 1.666\times10^{-7}\,\text{H/m} \quad \Rightarrow \quad \underline{\underline{L = 166.6\ \text{nH/m}}}$$

$$G = \frac{2\pi\sigma}{\ln\ b/a} = \frac{2\pi\sigma_{\text{air}}}{\ln\ b/a} = \frac{2\pi\,(0)}{\ln\ 2.3/1} \quad \Rightarrow \quad \underline{\underline{G=0}}$$

$$C = \frac{2\pi\varepsilon}{\ln\ b/a} = \frac{2\pi\,(8.854\times10^{-12})}{\ln\ 2.3/1} = 6.679\times10^{-11}\ \text{F/m} \quad \Rightarrow \quad \underline{\underline{C = 66.79\ \text{pF/m}}}$$

$$= \frac{\mu\varepsilon}{L} = \frac{4\pi\times10^{-7}\,(8.854\times10^{-12})}{1.666\times10^{-7}} = 6.679\times10^{-11}\ \text{F/m (same)}$$

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$$\beta = \omega\sqrt{LC} = 2\pi(1\times10^{9})\sqrt{1.666\times10^{-7}(6.679\times10^{-11})} \quad \Rightarrow \quad \underline{\underline{\beta = 20.959\ \text{rad/m}}}$$

$$u = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.666\times10^{-7}(6.679\times10^{-11})}} \quad \Rightarrow \quad \underline{\underline{u = 2.998\times10^{8}\ \text{m/s}}}$$

$$\lambda = 2\pi/\beta = u/f = 2.998\times10^{8}/1\times10^{9} \quad \Rightarrow \quad \underline{\underline{\lambda = 0.2998\ \text{m}}}$$

$$Z_0 = \sqrt{L/C} = \sqrt{1.666\times10^{-7}\ /\ 6.679\times10^{-11}} \quad \Rightarrow \quad \underline{\underline{Z_0 = 49.94\ \Omega}}$$