

We want to implement an impedance equivalent to that of a 2 nH inductor at a frequency of 3 GHz using the shortest possible stub made from 50 Ω microstrip transmission line ($u = 2 \times 10^8$ m/s) terminated in either a short or open circuit.

Find phase constant:
$$\beta = \frac{\omega}{u} = \frac{2\pi f}{u} = \frac{2\pi(3 \times 10^9)}{2 \times 10^8} \Rightarrow \underline{\beta = 94.24778 \text{ rad/m}}$$

Find 2nH inductor impedance:

$$Z_{2\text{nH}} = j\omega L = j2\pi f L = j2\pi(3 \times 10^9)2 \times 10^{-9} \Rightarrow \underline{Z_{2\text{nH}} = j37.699 \Omega}$$

Determine shortest possible length for a short circuit stub:

$$Z_{SC} = Z_{2\text{nH}}$$

$$jZ_0 \tan(\beta l_{sc}) = j50 \tan(94.2478 l_{sc}) = j37.699 \Omega$$

$$l_{sc} = \frac{\tan^{-1}(37.699 / 50)}{94.2478} \Rightarrow \underline{l_{sc} = 6.855 \text{ mm}}$$

Determine shortest possible length for an open circuit stub:

$$Z_{OC} = Z_{2\text{nH}}$$

$$\frac{Z_0}{j \tan(\beta l_{oc})} = \frac{50}{j \tan(94.2478 l_{oc})} = j37.699 \Omega$$

$$l_{oc} = \frac{\tan^{-1}(-j50 / j37.399)}{94.2478} \Rightarrow l_{oc} = -9.812 \text{ mm (not possible!?!)}$$

Remember that one can always add integer multiples of half wavelength to transmission line lengths and get same impedance. Here the wavelength is:

$$\lambda = \frac{u}{f} = \frac{2 \times 10^8}{3 \times 10^9} = 66.\bar{6} \text{ mm} \Rightarrow \underline{\lambda / 2 = 33.\bar{3} \text{ mm}}$$

So, the shortest possible realizable open circuit stub would have length:

$$l_{oc} = -9.812 \text{ mm} + (1)(\lambda / 2) = -9.812 + 33.\bar{3} \Rightarrow \underline{l_{oc} = 23.521 \text{ mm}}$$

Short circuit stub wins! However, both are equivalent to a 2 nH inductor @ 3 GHz.

