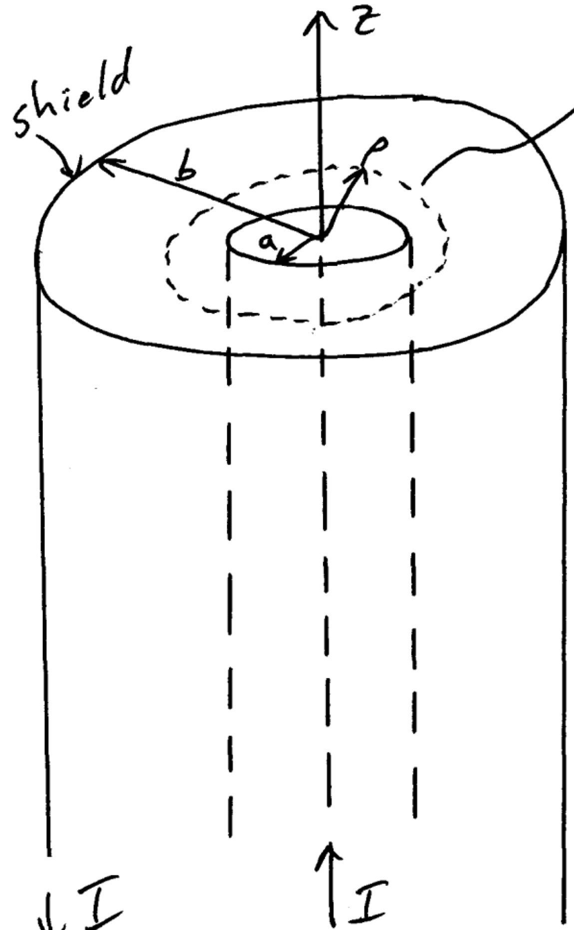
Example- Find the magnetic field everywhere for a long coaxial transmission line with solid inner conductor of radius *a* and shield of radius *b* carrying a uniformly distributed current *I* through the inner conductor in the +*z*-direction and uniformly distributed current -*I* on the shield in the -*z*-direction.



Amperian contour is circle of radius ρ at constant z. Based on circular cylindrical symmetry, we expect $\bar{H} = \hat{a}_\phi H_\phi$.

<u>Ampere's Law</u>

$$\oint_C \bar{H}\cdot d\bar{l} = I_{enc}$$

For the LHS of equation -

$$\oint_{circle} H_\phi \hat{a}_\phi \cdot (\cancel{d\rho}^{\,0}\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + \cancel{dz}^{\,0}\, \hat{a}_z)$$

$$= \oint_{circle} H_\phi \rho\, d\phi = \underline{H_\phi (2\pi\rho)}$$

The RHS of the equation can be broken down into three regions based on how much current is enclosed.

<u>Region 1 $0<\rho<a$</u> → contour encloses only portion of I

current density $\bar{J} = \hat{a}_z \frac{I}{\pi a^2}$

$$I_{enc} = \iint_S \bar{J}\cdot d\bar{s} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \hat{a}_z \frac{I}{\pi a^2}\cdot \hat{a}_z\, \rho\, d\rho\, d\phi = \frac{I}{\pi a^2}(2\pi)\frac{\rho^2}{2}$$

$$= I\, \rho^2/a^2$$

Region 1 $0 < \rho < a$ cont.

$$\oint_c \bar{H} \cdot d\bar{\ell} = I_{enc}$$

$$H_\phi (2\pi\rho) = I \frac{\rho^2}{a^2} \Rightarrow H_\phi = \frac{I\rho}{2\pi a^2}$$

$$\underline{\bar{H} = \hat{a}_\phi \frac{I\rho}{2\pi a^2}}$$

Region 2 $a < \rho < b$ → contour encloses entire inner conductor

$$\oint_c \bar{H} \cdot d\bar{\ell} = H_\phi (2\pi\rho) = I_{enc} = I$$

$$\hookrightarrow H_\phi = \frac{I}{2\pi\rho} \Rightarrow \underline{\bar{H} = \hat{a}_\phi \frac{I}{2\pi\rho}}$$

Region 3 $\rho > b$ → contour encloses both inner conductor and shield

$$\oint_c \bar{H} \cdot d\bar{\ell} = H_\phi (2\pi\rho) = I_{enc} = I - I = 0$$

$$\hookrightarrow H_\phi = 0 \Rightarrow \underline{\bar{H} = 0}$$

$$\boxed{\bar{H} = \begin{cases} \hat{a}_\phi \dfrac{I\rho}{2\pi a^2} & 0 < \rho < a \\ \hat{a}_\phi \dfrac{I}{2\pi\rho} & a < \rho < b \\ 0 & \rho > b \end{cases}}$$