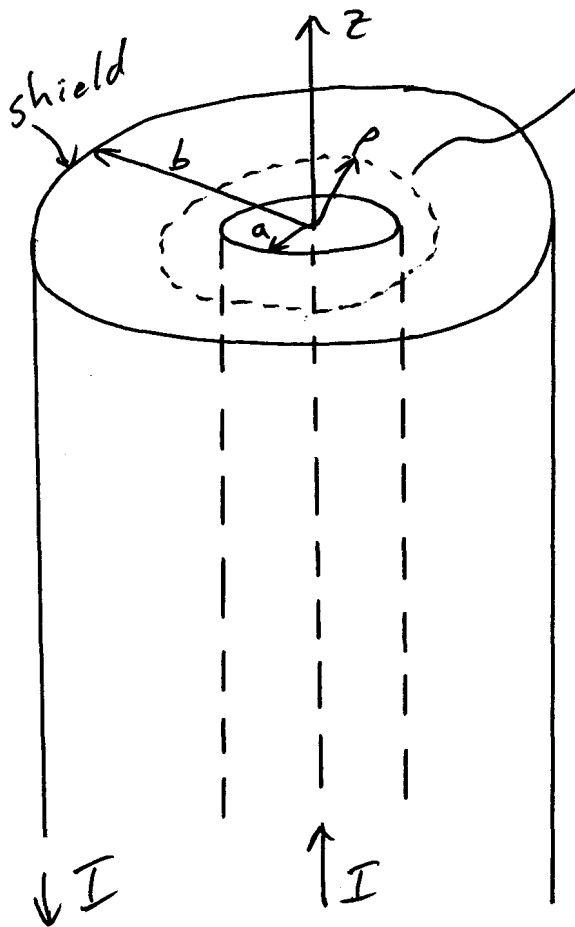


**Example-** Find the magnetic field everywhere for a long coaxial transmission line with solid inner conductor of radius  $a$  and shield of radius  $b$  carrying a uniformly distributed current  $I$  in the inner conductor in the  $+z$ -direction and uniformly distributed current  $-I$  on the shield in the  $-z$ -direction.



Amperian contour is circle of radius  $\rho$  at constant  $z$ . Based on circular cylindrical symmetry, we expect  $\vec{H} = \hat{a}_\phi H_\phi$ .

Ampere's Law

$$\oint_c \vec{H} \cdot d\vec{l} = I_{enc}$$

For the LHS of equation -

$$\oint_{\text{circle}} H_\phi \hat{a}_\phi \cdot (d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + d\phi \hat{a}_z)$$

$$= \oint_{\text{circle}} H_\phi \rho d\phi = \underline{H_\phi (2\pi\rho)}$$

The RHS of the equation can be broken down into three regions based on how much current is enclosed.

Region 1  $0 < \rho \leq a$   $\rightarrow$  contour encloses only portion of  $I$

$$\text{current density } \vec{J} = \hat{a}_z \frac{I}{\pi a^2}$$

$$\begin{aligned} I_{enc} &= \iint_S \vec{J} \cdot d\vec{s} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\rho} \hat{a}_z \frac{I}{\pi a^2} \cdot \hat{a}_z \rho d\rho d\phi = \frac{I}{\pi a^2} (2\pi) \frac{\rho^2}{2} \\ &= I \frac{\rho^2}{a^2} \end{aligned}$$

Region 1  $0 < \rho \leq a$  cont.

$$\oint_c \vec{H} \cdot d\vec{\ell} = I_{enc}$$

$$H_\phi (2\pi\rho) = I \frac{\rho^2}{a^2} \Rightarrow H_\phi = \frac{I\rho}{2\pi a^2}$$

$$\underline{\vec{H} = \hat{a}_\phi \frac{I\rho}{2\pi a^2}}$$

Region 2  $a \leq \rho < b$   $\rightarrow$  contour encloses entire inner conductor

$$\oint_c \vec{H} \cdot d\vec{\ell} = H_\phi (2\pi\rho) = I_{enc} = I$$

$$\hookrightarrow H_\phi = \frac{I}{2\pi\rho} \Rightarrow \underline{\vec{H} = \hat{a}_\phi \frac{I}{2\pi\rho}}$$

Region 3  $\rho > b$   $\rightarrow$  contour encloses both inner conductor and shield

$$\oint_c \vec{H} \cdot d\vec{\ell} = H_\phi (2\pi\rho) = I_{enc} = I - I = 0$$

$$\hookrightarrow H_\phi = 0 \Rightarrow \underline{\vec{H} = 0}$$

$$\vec{H} = \begin{cases} \hat{a}_\phi \frac{I\rho}{2\pi a^2} & 0 < \rho \leq a \\ \hat{a}_\phi \frac{I}{2\pi\rho} & a \leq \rho < b \\ 0 & \rho > b \end{cases}$$