# EE 362 Electronic, Magnetic, & Optical Properties of Materials
# Examination #1 (Spring 2026)

**Name** KEY

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class & show all work for credit. Insert equation sheet in exam. Answers should have **5-6 significant figures** (use more for constants).

1) At 300 K, calculate the number of hole energy states (#/cm$^3$) in Germanium between $E_v$ and $E_v - 1.3k_BT$. Find the probability that an energy state at $E_v$ is occupied by a hole. Next, find the probability that an energy state at $E_v - 1.3k_BT$ is occupied by a hole. Assume the Fermi energy is at mid bandgap.

Per (3.75), $g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$. Following a process similar to Example 3.4, the number of valence energy states in Germanium between $E_v$ and $E_v$ - $1.3k_BT$ is

$$N_v = \int_{E_v-1.3k_BT}^{E_v} g_v(E)\,dE = \int_{E_v-1.3k_BT}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}\,dE$$

$$= \left[\frac{4\pi(2m_p^*)^{3/2}}{h^3}\frac{-2}{3}(E_v - E)^{3/2}\right]\Bigg|_{E_v-1.3k_BT}^{E_v} = \frac{8\pi(2m_p^*)^{3/2}}{3h^3}(1.3k_BT)^{3/2}.$$

Per Table B.4, the density of states effective hole mass for Germanium is $m_p^* = 0.37m_0$. Using this value and other constants, we get

$$N_v = \frac{8\pi\left(2(0.37)9.1093837\times10^{-31}\right)^{3/2}}{3\left(6.62607015\times10^{-34}\right)^3}\left[1.3(1.380649\times10^{-23})\,300\right]^{3/2}$$

$\Rightarrow$ **$N_v = 6.297238\times10^{24}$ #/m$^3$ = $6.297238\times10^{18}$ #/cm$^3$.**

Per example 3.7, the probability that an energy state is empty at $E$ can be found by subtracting the probability of electron at that energy from 1. To compute this, we need

$\Delta E_1 = E_F - E_v = E_{gap}/2 = 0.66/2 =$ **0.33 eV** & $k_BT = (8.617333\times10^{-5})300 =$ **0.025852 eV**

$$\text{P\_hole\_E1} = 1 - f_F(E_v) = 1 - \frac{1}{1+e^{\frac{E_v - E_F}{k_BT}}} = 1 - \frac{1}{1+e^{\frac{-\Delta E_1}{k_BT}}} = 1 - \frac{1}{1+e^{\frac{-0.33}{0.025852}}} = \mathbf{2.859187\times10^{-6}}$$

$\Delta E_2 = E_F - (E_v - 1.3k_BT) = E_{gap}/2 + 1.3k_BT = 0.33 + 1.3(0.025852) =$ **0.363608 eV**

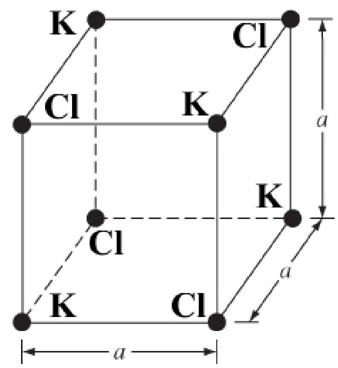
$$\text{P\_hole\_E2} = 1 - f_F(E_v - 1.3k_BT) = 1 - \frac{1}{1+e^{\frac{E_v - 1.3k_BT - E_F}{k_BT}}} = 1 - \frac{1}{1+e^{\frac{-\Delta E_2}{k_BT}}} = 1 - \frac{1}{1+e^{\frac{-0.363608}{0.025852}}}$$

$= \mathbf{7.79221\times10^{-7}}$.

# of hole energy states$_{E_v \text{ to } E_v - 1.3k_BT}$ = **$6.297238\times10^{24}$ #/m$^3$ = $6.297238\times10^{18}$ #/cm$^3$**

hole prob.$_{E_v}$ = **$2.85919\times10^{-6}$** hole prob.$_{E_v - 1.3k_BT}$ = **$7.79221\times10^{-7}$**

2) The crystal structure of potassium chloride (KCl) is simple cubic with the K (ionic effective radius 1.341 Å) and Cl (ionic effective radius 4.951 Å) atoms alternating. Find the lattice constant (Å), unit cell volume ($m^3$), # of K atoms per unit cell, and # Cl atoms per unit cell. Then, compute the atomic volume density *avd* (#/$m^3$) for the K and Cl atoms. Last, find the mass density of KCl (g/$cm^3$).




Assuming the K and Cl atoms touch at their effective radius-

$a = r_K + r_{Cl} = 1.341 + 4.951 =$ **6.292 Å**

unit cell volume = $a^3 = (6.292\times10^{-10})^3 =$ **$2.4909565 \times 10^{-28}$ m$^3$**

# K atoms/unit cell = 4 K atoms at corners (1/8 atom/corner) = **0.5**

# Cl atoms/unit cell = 4 Cl atoms at corners (1/8 atom/corner) = **0.5**

$$avd_K = \frac{\text{\# K atoms/unit cell}}{\text{volume/unit cell}} = \frac{0.5}{2.4909565\times10^{-28}} \Rightarrow \boldsymbol{avd_K} = \mathbf{2.007261 \times 10^{27}\ \#/m^3}.$$

$$avd_{Cl} = \frac{\text{\# Cl atoms/unit cell}}{\text{volume/unit cell}} = \frac{0.5}{2.4909565\times10^{-28}} \Rightarrow \boldsymbol{avd_{Cl}} = \mathbf{2.007261 \times 10^{27}\ \#/m^3}.$$

From the periodic table, the atomic weight of potassium K is 39.098 and the atomic weight of chlorine Cl is 35.453.

$$md_{KCl} = \frac{\text{atomic weight K}\,(avd_K) + \text{atomic weight Cl}\,(avd_{Cl})}{N_A}$$

$$= \frac{39.098\,(2.007261\times10^{27}) + 35.453\,(2.007261\times10^{27})}{6.02214076\times10^{23}}$$

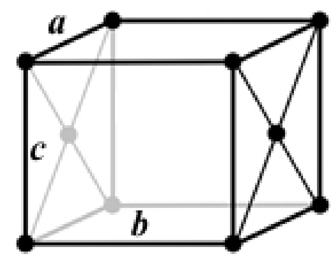
$$\Rightarrow \boldsymbol{md_{KCl}} = \mathbf{248{,}488.5795\ g/m^3} = \mathbf{0.248489\ g/cm^3}.$$

KCl lattice constant = **6.292 Å** unit cell volume = **$2.4909565 \times 10^{-28}$ m$^3$**

# of K atoms per unit cell = **0.5** # of Cl atoms per unit cell = **0.5**

$avd_K$ = **$2.00726 \times 10^{27}$ #/m$^3$** $avd_{Cl}$ = **$2.00726 \times 10^{27}$ #/m$^3$** mass density$_{KCl}$ = **0.248489 g/cm$^3$**

3) A mineral has a side-centered orthorhombic crystal lattice where $a$ = 2.6 Å, $b$ = 5.2 Å, and $c$ = 3.8 Å. Find the number of atoms per unit cell and atomic **volume** density. Then, calculate the number of atoms and atomic **surface** density for the right & top surfaces.




# atoms/unit cell = 8 atoms at corners (1/8 atom/corner) + 2 atoms at sides (1/2 atom/side)

$\Rightarrow$ # atoms/unit cell = **2**

unit cell volume = $abc = (2.6\times10^{-10})(5.2\times10^{-10})(3.8\times10^{-10})$ = $\mathbf{5.1376 \times 10^{-29}\ m^3}$

$$avd = \frac{\text{\# atoms/unit cell}}{\text{volume/unit cell}} = \frac{2}{5.1376\times10^{-22}}$$

$\Rightarrow$ ***avd*** = **$3.89287\times10^{28}$ #/m³** = **$3.89287\times10^{28}$ #/m³**.

## Right side

unit cell right side area = $ac = (2.6\times10^{-10})(3.8\times10^{-10})$ = $\mathbf{9.88 \times 10^{-20}\ m^2}$

# atoms/right side = 4 atoms at corners (1/4 atom/corner) + 1 atom in middle

$\Rightarrow$ **# atoms/right side = 2**

$$asd_R = \frac{\text{\# atoms/right side}}{\text{area/right side}} = \frac{2}{9.88\times10^{-20}}$$

$\Rightarrow$ ***$asd_R$*** = **$2.02429\times10^{19}$ #/m²** = **$2.02429\times10^{15}$ #/cm²**.

## Top side

unit cell top area = $ab = (2.6\times10^{-10})(5.2\times10^{-10})$ = $\mathbf{1.352 \times 10^{-19}\ m^2}$

# atoms/top = 4 atoms at corners (1/4 atom/corner) $\Rightarrow$ **# atoms/top = 1**

$$asd_T = \frac{\text{\# atoms/top}}{\text{area/top}} = \frac{1}{1.352\times10^{-19}}$$ $\Rightarrow$ ***$asd_T$*** = **$7.39645\times10^{18}$ #/m²** = **$7.39645\times10^{14}$ #/cm²**.

# atoms/unit cell = **2** atomic vol. density = **$3.89287\times10^{28}$ #/m³** = **$3.89287\times10^{22}$ #/cm³**

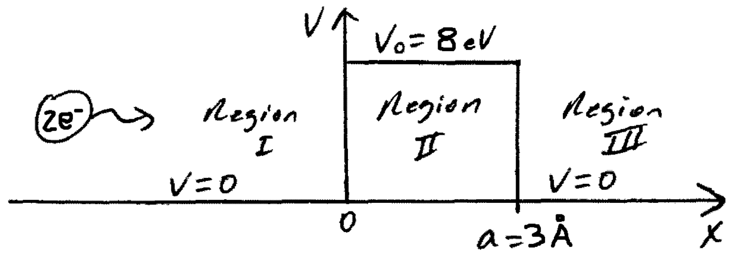
# $\text{atoms}_{right}$ = **2** atomic surface $\text{density}_{right}$ = **$2.02429\times10^{19}$ #/m²** = **$2.02429\times10^{15}$ #/cm²**

# $\text{atoms}_{top}$ = **1** atomic surface $\text{density}_{top}$ = **$7.39645\times10^{18}$ #/m²** = **$7.39645\times10^{14}$ #/cm²**

4) An eccentric scientist has fused two electrons to create the bielectron. The bielectron is accelerated from rest in the +$x$-direction through a 3.3 V potential difference at $x \to -\infty$. Find the kinetic energy (J and eV), momentum, velocity, and de Broglie wavelength of the bielectron assuming region I ($x < 0$) has zero potential energy. The bielectron encounters a finite potential barrier of 8 eV from $0 \le x \le 3$ Å (region II). Find the wave number $k$ in regions I & II as well as the percent transmission coefficient (AKA tunneling probability) for the bielectron from region I to region III ($x > 0$).

Mass of bielectron = $2m_0$

Magnitude of charge of bielectron = $2e$




From EE 381, the kinetic energy is $E = qV = 2e(3.3\text{ V})$ $\Rightarrow$ **$E$ = 6.6 eV = 1.0574366 × 10$^{-18}$ J**.

From physics, the kinetic energy is $E = 0.5mv^2$. The bielectron velocity is then

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1.0574366\times10^{-18})}{2(9.1093837015\times10^{-31})}} \quad \Rightarrow \mathbf{v = 1{,}077{,}414.1\ m/s = 1.07741\times10^{6}\ m/s}.$$

From physics, the momentum is $p = mv = 2(9.1093837015\times10^{-31}\text{ kg})\ 1{,}077{,}414.1\text{ m/s}$

$\Rightarrow$ **$p$ = 1.962916 × 10$^{-24}$ kg-m/s**.

Per (2.3), the de Broglie wavelength is

$\lambda = h/p = 6.62607015\times10^{-34}/1.962916\times10^{-24}$ $\Rightarrow$ **$\lambda$ = 3.37563 × 10$^{-10}$ m = 3.37563 Å**.

Per (2.61a & 2.61b), the wave numbers in regions I & II are

$$k_I = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{(2)2(9.1093837\times10^{-31})1.0574366\times10^{-18}}{(1.0545718\times10^{-34})^2}} \quad \Rightarrow \mathbf{k_I = 1.86134\times10^{10}\ rad/m}.$$

$$k_{II} = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} = \sqrt{\frac{(2)2(9.1093837\times10^{-31})[8(1.6021766\times10^{-19})-1.0574366\times10^{-18}]}{(1.0545718\times10^{-34})^2}}$$

$\Rightarrow$ **$k_{II}$ = 8.572698 × 10$^{9}$ Np/m**.

% transmission coeff. is
$$T_{exact} = \frac{1}{1+\dfrac{V_0^2\sinh^2(k_{II}a)}{4E(V_0-E)}} = \frac{1}{1+\dfrac{8^2\sinh^2(8.572698\times10^{9}\bullet3\times10^{-10})}{4(6.6)(8-6.6)}}$$

$\Rightarrow$ **$T_{exact}$ = 0.01345757 = 1.345757 %**.

kinetic energy = **6.6 eV** = **1.05744 × 10$^{-18}$ J** momentum = **1.962916 × 10$^{-24}$ kg-m/s**

velocity = **1.07741 × 10$^{6}$ m/s** de Broglie wavelength = **3.37563 × 10$^{-10}$ m = 3.37563 Å**

$k_I$ = **1.86134 × 10$^{10}$ rad/m** $k_{II}$ = **8.572698 × 10$^{9}$ Np/m** % trans. coeff. = **1.34576 %**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 H Hydrogen 1.008 | | | | | | | | | | | | | | | | | 2 He Helium 4.003 |
| 3 Li Lithium 6.941 | 4 Be Beryllium 9.012 | | | | | | | | | | | 5 B Boron 10.811 | 6 C Carbon 12.011 | 7 N Nitrogen 14.007 | 8 O Oxygen 15.999 | 9 F Fluorine 18.998 | 10 Ne Neon 20.180 |
| 11 Na Sodium 22.990 | 12 Mg Magnesium 24.305 | | | | | | | | | | | 13 Al Aluminum 26.982 | 14 Si Silicon 28.086 | 15 P Phosphorus 30.974 | 16 S Sulfur 32.066 | 17 Cl Chlorine 35.453 | 18 Ar Argon 39.948 |
| 19 K Potassium 39.098 | 20 Ca Calcium 40.078 | 21 Sc Scandium 44.956 | 22 Ti Titanium 47.88 | 23 V Vanadium 50.942 | 24 Cr Chromium 51.996 | 25 Mn Manganese 54.938 | 26 Fe Iron 55.845 | 27 Co Cobalt 58.933 | 28 Ni Nickel 58.693 | 29 Cu Copper 63.546 | 30 Zn Zinc 65.38 | 31 Ga Gallium 69.723 | 32 Ge Germanium 72.631 | 33 As Arsenic 74.922 | 34 Se Selenium 78.971 | 35 Br Bromine 79.904 | 36 Kr Krypton 83.798 |
| 37 Rb Rubidium 85.468 | 38 Sr Strontium 87.62 | 39 Y Yttrium 88.906 | 40 Zr Zirconium 91.224 | 41 Nb Niobium 92.906 | 42 Mo Molybdenum 95.95 | 43 Tc Technetium 98.907 | 44 Ru Ruthenium 101.07 | 45 Rh Rhodium 102.906 | 46 Pd Palladium 106.42 | 47 Ag Silver 107.868 | 48 Cd Cadmium 112.414 | 49 In Indium 114.818 | 50 Sn Tin 118.711 | 51 Sb Antimony 121.760 | 52 Te Tellurium 127.6 | 53 I Iodine 126.904 | 54 Xe Xenon 131.294 |
| 55 Cs Cesium 132.905 | 56 Ba Barium 137.328 | 57-71 | 72 Hf Hafnium 178.49 | 73 Ta Tantalum 180.948 | 74 W Tungsten 183.85 | 75 Re Rhenium 186.207 | 76 Os Osmium 190.23 | 77 Ir Iridium 192.22 | 78 Pt Platinum 195.08 | 79 Au Gold 196.967 | 80 Hg Mercury 200.59 | 81 Tl Thallium 204.383 | 82 Pb Lead 207.2 | 83 Bi Bismuth 208.980 | 84 Po Polonium [208.982] | 85 At Astatine 209.987 | 86 Rn Radon 222.018 |
| 87 Fr Francium 223.020 | 88 Ra Radium 226.025 | 89-103 | 104 Rf Rutherfordium [261] | 105 Db Dubnium [262] | 106 Sg Seaborgium [266] | 107 Bh Bohrium [264] | 108 Hs Hassium [269] | 109 Mt Meitnerium [278] | 110 Ds Darmstadtium [281] | 111 Rg Roentgenium [280] | 112 Cn Copernicium [285] | 113 Nh Nihonium [286] | 114 Fl Flerovium [289] | 115 Mc Moscovium [289] | 116 Lv Livermorium [293] | 117 Ts Tennessine [294] | 118 Og Oganesson [294] |

**Table B.4** | Silicon, gallium arsenide, and germanium properties ($T = 300$ K)

| Property | Si | GaAs | Ge |
|---|---|---|---|
| Atoms (cm$^{-3}$) | $5.0 \times 10^{22}$ | $4.42 \times 10^{22}$ | $4.42 \times 10^{22}$ |
| Atomic weight | 28.09 | 144.63 | 72.60 |
| Crystal structure | Diamond | Zincblende | Diamond |
| Density (g/cm$^3$) | 2.33 | 5.32 | 5.33 |
| Lattice constant (Å) | 5.43 | 5.65 | 5.65 |
| Melting point (°C) | 1415 | 1238 | 937 |
| Dielectric constant | 11.7 | 13.1 | 16.0 |
| Bandgap energy (eV) | 1.12 | 1.42 | 0.66 |
| Density of states effective mass | | | |
| Electrons $\left(\frac{m^*_{dn}}{m_o}\right)$ | 1.08 | 0.067 | 0.55 |
| Holes $\left(\frac{m^*_{dp}}{m_o}\right)$ | 0.56 | 0.48 | 0.37 |