EE 362 Electronic, Magnetic, & Optical Properties of Materials
Examination #3 (Spring 2025)

Name __KEY A__

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class & **show all** work for credit. Insert equation sheet in exam. Answers should have **4-5 significant figures** (use more for constants).

1) For a uniformly doped ($N_a = 2\times10^{16}$ cm^{-3} on the p-side and $N_d = 6\times10^{16}$ cm^{-3} on the n-side) germanium ($\varepsilon_{r,\mathrm{Ge}} = 16.1$, $n_i = 4\times10^{12}$ cm^{-3}) pn junction at **270 K** with cross-sectional area 50×10^{-9} m^2, calculate V_{bi}, x_n, x_p, W, C', and C when the junction is reverse biased to 1.2 V.

Given: $n_i = 4\times10^{12}$ cm^{-3} and $\varepsilon_r = 16.1$ for germanium at 270 K.

Per (7.10), $$V_{bi} = \frac{k_B T}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = \frac{8.617333\bullet10^{-7}\ \text{eV/K}\ (270\ \text{K})}{e}\ln\left(\frac{2\bullet10^{16}(6\bullet10^{16})}{(4\bullet10^{12})^2}\right)$$

$\Rightarrow$ __V_{bi} = 0.421897 V__.

Per (7.28) w/ $V_{bi} \to V_{bi} + V_R = 0.4219 + 1.2 = 1.6219$ V,

$$x_n = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a+N_d}\right\}^{1/2} = \left\{\frac{2(16.1)8.8541878\bullet10^{-12}(1.6219)}{1.602176634\bullet10^{-19}}\left(\frac{2}{6}\right)\frac{1}{2\bullet10^{22}+6\bullet10^{22}}\right\}^{1/2}$$

$\Rightarrow$ __$x_n = 1.096612\times10^{-7}$ m = 109.6612 nm__.

Per (7.29) w/ $V_{bi} \to V_{bi} + V_R = 1.6219$ V,

$$x_p = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\frac{1}{N_a+N_d}\right\}^{1/2} = \left\{\frac{2(16.1)8.8541878\bullet10^{-12}(1.6219)}{1.602176634\bullet10^{-19}}\left(\frac{6}{2}\right)\frac{1}{2\bullet10^{22}+6\bullet10^{22}}\right\}^{1/2}$$

$\Rightarrow$ __$x_p = 3.289837\times10^{-7}$ m = 328.9837 nm__.

$W = x_p + x_n = 1.096612\times10^{-7} + 3.28937\times10^{-7}$ $\Rightarrow$ __$W = 4.386449\times10^{-7}$ m = 438.6449 nm__.

Per (7.43), $C' = \varepsilon_s / W = 16.1\,(8.85419\times10^{-12}) / 4.386449\times10^{-7}$

$\Rightarrow$ __$C' = 3.24984\times10^{-4}$ F/m^2 = 3.24984×10^{-8} F/cm^2__.

$C = A\,C' = (50\times10^{-9})\,3.2498\times10^{-4}$ $\Rightarrow$ __C = 16.2492 pF__

V_{bi} = __0.4219 V__ x_n = __1.09661×10^{-7} m = 109.661 nm__ x_p = __3.28984×10^{-7} m = 328.984 nm__

W = __4.38645×10^{-7} m = 438.645 nm__ C' = __3.2498×10^{-4} F/m^2__ C = __16.2492 pF__

2) For a uniformly doped ($N_a = 2 \times 10^{15}$ cm^{-3} on p-side and $N_d = 6 \times 10^{15}$ cm^{-3} on n-side, $D_n = 200$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_{n0} = 50$ ns, & $\tau_{p0} = 10$ ns) GaAs pn junction at **300 K**, calculate the majority n_{n0} and minority p_{n0} thermal equilibrium carrier concentrations (cm^{-3}) on the **n-side**. Then, calculate the minority excess carrier concentration p_n (cm^{-3}) at the boundary between the depletion layer and n-side when a forward bias of 0.62 V is applied. Comparing p_n and n_{n0}, is the low-level injection assumption for the ambipolar transport equation justified? Why? Comparing p_n and p_{n0}, is the thermal equilibrium hole concentration irrelevant/rounding error or is it relevant/appreciable fraction? In the n- & p-regions, assume the electric field is negligible, no additional excess carriers are being generated, and enough time has elapsed for the pn junction to be at steady-state. Applying the conditions under which the pn junction is operating, write the **particular** ambipolar transport differential equation for excess minority carriers on the **n-side** in standard form with all constant coefficients evaluated. **Extra credit:** Assuming the boundary is at $x = 0$ and that the n-side is $x > 0$, find the excess minority charge carrier concentration (cm^{-3}) as a function of x on the n-side of the junction.

From Table B.4, $n_i = 1.8 \times 10^6$ cm^{-3} = 1.8×10^{12} m^{-3} and $\varepsilon_r = 13.1$ for GaAs at 300 K.

Per (7.10), $V_t = \dfrac{8.617333 \bullet 10^{-7}\ \text{eV/K}\,(300\,\text{K})}{e} = 0.025852\ \text{V}$

n-side (majority carriers are electrons)

$n_{n0} \approx N_d$ $\Rightarrow$ **$n_{n0} = 6 \times 10^{15}$ cm^{-3}**.

Per (4.43), $p_{n0} = n_i^2/n_{n0} = (1.8 \times 10^6)^2/6 \times 10^{15}$ $\Rightarrow$ **$p_{n0} = 5.4 \times 10^{-4}$ cm^{-3} = 0.00054 cm^{-3}**.

Using (8.7), $p_n = p_{n0} e^{V_a/V_t} = 0.00054 e^{0.62/0.025852}$ $\Rightarrow$ **$p_n = 1.40584 \times 10^7$ cm^{-3}**.

Comparing p_n and n_{n0}, is the low-level injection assumption … justified?

$\Rightarrow$ **YES, $p_n = 1.40584 \times 10^7$ cm^{-3} << $n_{n0} = 6 \times 10^{15}$ cm^{-3}.**

Comparing p_n & p_{n0}, is p_{n0} irrelevant/rounding error … relevant/appreciable fraction?

$\Rightarrow$ **Since $p_n = 1.406 \times 10^7$ cm^{-3} >> $p_{n0} = 5.4 \times 10^{-4}$ cm^{-3}, p_{n0} is irrelevant/rounding error.**

n_{n0} = **6×10^{15} cm^{-3}** p_{n0} = **5.4×10^{-4} cm^{-3} = 0.00054 cm^{-3}** p_n = **1.40584×10^7 cm^{-3}**

2) continued

Per (6.56), for n-type semiconductor, the general ambipolar transport equation is

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}.$$

'assume the electric field is negligible' $\Rightarrow E \approx 0$

'no additional excess carriers are being generated' $\Rightarrow g' \approx 0$

'enough time has elapsed for the pn junction to be at steady-state' $\Rightarrow \frac{\partial(\delta p)}{\partial t} = 0$.

This leaves a particular ambipolar transport equation of

$$\frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{D_p \tau_{p0}} = \frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{0.001\,(10 \bullet 10^{-9})} = 0 \quad \Rightarrow \quad \frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{1 \bullet 10^{-11}} = 0.$$

Low-level injection assumption justified? **Yes** / No (circle correct)

Why? $\mathbf{p_n = 1.40584 \times 10^7\ cm^{-3} << n_{n0} = 6 \times 10^{15}\ cm^{-3}}$

Comparing p_n & p_{n0}, is p_{n0} **irrelevant/rounding error** or relevant/appreciable fraction? (circle correct)

Why? $\mathbf{p_n = 1.406 \times 10^7\ cm^{-3} >> p_{n0} = 5.4 \times 10^{-4}\ cm^{-3}}$

particular ambipolar transport eq'n = $\frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{1 \bullet 10^{-11}} = \frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{\left(3.1622776 \bullet 10^{-6}\right)^2} = 0$

Per (8.12), the solution to this wave equation is $\delta p_n(x) = A\, e^{x/3.1622776 \bullet 10^{-6}} + B\, e^{-x/3.1622776 \bullet 10^{-6}}$.

Boundary condition 1

$\delta p_n(x)$ must be finite for $x > 0$ $\quad \Rightarrow \quad A = 0$.

Boundary condition 2

$\delta p_n(x=0) = p_n(x=0) - p_{n0} \simeq 1.40584 \times 10^7 = B\, e^0 \quad \Rightarrow \quad B = 1.40584 \times 10^7\ \text{cm}^{-3}$.

Extra Credit:

excess minority charge carrier conc. = $\boldsymbol{\delta p_n(x) = 1.40584 \times 10^7\, e^{-x/3.1622776\text{e-}6}\ \text{cm}^{-3}\ \text{for } x > 0}$

3) At thermal equilibrium @ **290 K**, a silicon pn diode has the parameters given below. Assume the junction is uniformly doped on each side. Calculate the thermal voltage. For the p- & n-regions, find the thermal equilibrium electron & hole concentrations (cm^{-3}), diffusion coefficients (cm^2/s), diffusion lengths (μm), and conductivities (S/m). Then, find the series resistance (Ω) of the p- & n-regions as well as the overall series resistance (Ω). Next, find the portion of the dc current due to the holes I_{p0} (A) and electrons I_{n0} (A) when a voltage of 0.47 V is applied. Use these currents to determine the diffusion conductance g_d (S), resistance r_d (Ω), and capacitance C_d (F).

Silicon diode	$A = 3 \times 10^{-4}$ cm^2	$n_i = 6.2 \times 10^9$ cm^{-3}	$\varepsilon_r = 11.7$	$E_g = 1.12$ eV
p-region	length = $7.6\,L_p$	$N_a = 3 \times 10^{16}$ cm^{-3}	$\mu_p = 390$ cm^2/V-s	$\tau_{p0} = 97$ ns
n-region	length = $4.8\,L_n$	$N_d = 8 \times 10^{15}$ cm^{-3}	$\mu_n = 1600$ cm^2/V-s	$\tau_{n0} = 800$ ns

Per (7.10), $V_t = \dfrac{k_B T}{e} = \dfrac{8.617333 \bullet 10^{-7}\ \text{eV/K}\,(290\,\text{K})}{e}$ $\Rightarrow$ **$V_t = 0.024990266$ V**.

p-region (majority carriers are holes)

$p_{p0} \approx N_a$ $\Rightarrow$ **$p_{p0} = 3 \times 10^{16}$ cm^{-3}**.

Using (4.43), $n_{p0} = n_i^2/p_{p0} = (6.2 \times 10^9)^2 / 3 \times 10^{16}$ $\Rightarrow$ **$n_{p0} = 1281.333$ cm^{-3}**.

Using Einstein relation (5.47), $\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = \dfrac{k_B T}{e} = 0.024990266$ V.

$D_p = 0.024990266(\mu_p) = 0.024990266(390)$ $\Rightarrow$ **$D_p = 9.746204$ cm^2/s**.

Per (8.9), $L_p = \sqrt{D_p \tau_{p0}} = \sqrt{9.746204 \bullet 10^{-4}(97 \bullet 10^{-9})}$ $\Rightarrow$ **$L_p = 9.72307 \times 10^{-6}$ m**.

Use (5.23), $\sigma = e(\mu_n n + \mu_p p)$ $\Rightarrow \sigma_p \simeq e\mu_p p_{p0}$ to get the conductivity (MKS units)-

$\sigma_p \approx 1.602176634 \times 10^{-19}\,(390 \times 10^{-4})\,3 \times 10^{22}$ $\Rightarrow$ **$\sigma_p = 187.45467$ S/m.**

n-region (majority carriers are electrons)

$n_{n0} \approx N_d$ $\Rightarrow$ **$n_{n0} = 8 \times 10^{15}$ cm^{-3}**.

Using (4.43), $p_{n0} = n_i^2/n_{n0} = (6.2 \times 10^9)^2 / 8 \times 10^{15}$ $\Rightarrow$ **$p_{n0} = 4805$ cm^{-3}**.

Using Einstein relation (5.47), $\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = \dfrac{k_B T}{e} = 0.024990266$ V.

$D_n = 0.024990266(\mu_n) = 0.024990266(1600)$ $\Rightarrow$ **$D_n = 39.984426$ cm^2/s**.

Per (8.10), $L_n = \sqrt{D_n \tau_{n0}} = \sqrt{39.98443 \bullet 10^{-4}(800 \bullet 10^{-9})}$ $\Rightarrow$ **$L_n = 5.65575 \times 10^{-5}$ m**.

Use (5.23), $\sigma = e(\mu_n n + \mu_p p)$ $\Rightarrow \sigma_n \simeq e\mu_n n_{n0}$ to get the conductivity (MKS units)-

$\sigma_n \approx 1.602176634 \times 10^{-19}\,(1600 \times 10^{-4})\,8 \times 10^{21}$ $\Rightarrow$ **$\sigma_n = 205.07861$ S/m.**

3) continued

Per (5.22b), $R = L/\sigma A$ for a conductive material. Using MKS units-

$$r_{s,\mathrm{p}} \approx L_{\mathrm{p\text{-}reg}}/(\sigma_p A) = 7.6(9.72307\times10^{-6})/[187.45467(3\times10^{-8})] \quad \Rightarrow \quad \underline{\mathbf{r_{s,p} = 13.14013\ \Omega}}.$$

$$r_{s,\mathrm{n}} \approx L_{\mathrm{n\text{-}reg}}/(\sigma_n A) = 4.8(5.65575\times10^{-5})/[205.0786(3\times10^{-8})] \quad \Rightarrow \quad \underline{\mathbf{r_{s,n} = 44.12554\ \Omega}}.$$

$$r_s = r_{s,\mathrm{p}} + r_{s,\mathrm{n}} = 13.14013 + 44.12554 \quad \Rightarrow \quad \underline{\mathbf{r_s = 57.26567\ \Omega}}.$$

Per (8.94), $$I_{p0} = \frac{eAD_p p_{n0}}{L_p} e^{V_0/V_t} = \frac{1.60217663\bullet10^{-19}(3\bullet10^{-8})9.7462\bullet10^{-4}(4805\bullet10^{6})}{9.72307\bullet10^{-6}} e^{0.47/0.02499}$$

$$\Rightarrow \underline{\mathbf{I_{p0} = 3.40779\times10^{-7}\ A}}.$$

Per (8.97),

$$I_{n0} = \frac{eAD_n n_{p0}}{L_n} e^{V_0/V_t} = \frac{1.60217663\bullet10^{-19}(3\bullet10^{-8})39.9844\bullet10^{-4}(1281.3\bullet10^{6})}{5.65575\bullet10^{-5}} e^{0.47/0.02499}$$

$$\Rightarrow \underline{\mathbf{I_{n0} = 6.4093\times10^{-8}\ A}}.$$

Per (8.104), $$g_d = \frac{1}{V_t}(I_{p0} + I_{n0}) = \frac{1}{0.02499}\left(3.4078\bullet10^{-7} + 6.4093\bullet10^{-8}\right)$$

$$\Rightarrow \underline{\mathbf{g_d = 1.62012\times10^{-5}\ S}}.$$

$$r_d = 1/g_d = 1/1.62012\times10^{-5} \quad \Rightarrow \underline{\mathbf{r_d = 6.17238\times10^{4}\ \Omega}}.$$

Per (8.105), $$C_d = \frac{1}{2V_t}(I_{p0}\tau_{p0} + I_{n0}\tau_{n0}) = \frac{\left[3.4078\bullet10^{-7}(97\bullet10^{-9}) + 6.4093\bullet10^{-8}(800\bullet10^{-9})\right]}{2(0.02499)}$$

$$\Rightarrow \underline{\mathbf{C_d = 1.68726\times10^{-12}\ F}}.$$

V_t = **0.0249903 V**

p-region:	p_{p0} = **3×10^{16} cm^{-3}**	n_{p0} = **1281.333 cm^{-3}**	D_p = **9.7462 cm^2/s**
	L_p = **9.723 µm**	σ_p = **187.455 S/m**	$r_{s,p}$ = **13.140 Ω**
n-region:	n_{n0} = **8×10^{15} cm^{-3}**	p_{n0} = **4805 cm^{-3}**	D_n = **39.9844 cm^2/s**
	L_n = **56.5575 µm**	σ_n = **205.079 S/m**	$r_{s,n}$ = **44.126 Ω**
	r_s = **57.266 Ω**	I_{p0} = **340.779 nA**	I_{n0} = **64.093 nA**
	g_d = **16.201 µS**	r_d = **61.724 kΩ**	C_d = **1.6873 pF**