EE 362 Electronic, Magnetic, & Optical Properties of Materials
Examination #1- Problem 4R (Spring 2025)

Name **KEY B**

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class & show all work for credit. Turn-in eq'n sheet with replacement problem. Answers should have 4-5 significant figures (use more for constants). Worth up to 20 points as replacement for original problem 4 (25 points).

1) A newly created 'skinny' electron ($m = 0.75m_0$) is confined in a 1D infinite potential well where $V = 0$ for $0 < x < a = 6.4$ Å and $V \to \infty$ elsewhere. For the first three possible states, find the wave number, kinetic energy (eV), and probability density function of the confined electron. For the third state, find the probability (%) of finding the electron in the range $0 < x < a/6$. [Hint: $\sin^2(A) = 0.5 - 0.5\cos(2A)$ and $\cos^2(A) = 0.5 + 0.5\cos(2A)$.]

Per (2.33), the wave number $k_n = n\pi/a$.

For $n = 1$, $k_1 = \pi/6.4 \times 10^{-10}$ $\Rightarrow$ **$k_1 = 4.908739 \times 10^9$ rad/m**.

For $n = 2$, $k_2 = 2\pi/6.4 \times 10^{-10}$ $\Rightarrow$ **$k_2 = 9.817477 \times 10^9$ rad/m**.

For $n = 3$, $k_3 = 3\pi/6.4 \times 10^{-10}$ $\Rightarrow$ **$k_3 = 1.472622 \times 10^{10}$ rad/m**.

Per (2.38), the kinetic energy

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054571817\times10^{-34})^2\,\pi^2\,n^2}{2(0.75)9.1093837\times10^{-31}(6.4\times10^{-10})^2}$$
$$= 1.961155\times10^{-19}\,n^2\ \text{(J)} = 1.22406\,n^2\ \text{(eV)}$$

For n =1, the level kinetic energy is $\Rightarrow$ **$E_1 = 1.96115 \times 10^{-19}$ J = 1.22406 eV**.

For n =2, the level kinetic energy is $\Rightarrow$ **$E_2 = 7.84462 \times 10^{-19}$ J = 4.89623 eV**.

For n =3, the level kinetic energy is $\Rightarrow$ **$E_3 = 1.76504 \times 10^{-18}$ J = 11.01651 eV**.

Per (2.35), the wave function is

$$\psi(x) = \sqrt{\frac{2}{a}}\sin(k_n x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)\quad (\text{m}^{-0.5})\quad n = 1, 2, 3\ldots$$

Per (2.17), the probability density function is

$$|\Psi(x,t)|^2 = |\psi(x)|^2 = \frac{2}{a}\sin^2\left(\frac{n\pi}{a}x\right) = \frac{2}{6.4\times10^{-10}}\sin^2\left(\frac{n\pi}{6.4\times10^{-10}}x\right)\quad (\text{m}^{-1}).$$

For n =1, we get **$|\Psi_1(x)|^2 = 3.125 \times 10^9 \sin^2(4.908739 \times 10^9 x)$ m^{-1}**.

For n =2, we get **$|\Psi_2(x)|^2 = 3.125 \times 10^9 \sin^2(9.817477 \times 10^9 x)$ m^{-1}**.

For n =3, we get **$|\Psi_3(x)|^2 = 3.125 \times 10^9 \sin^2(1.472622 \times 10^{10} x)$ m^{-1}**.

For the third state, the probability the 'skinny' electron is located $0 < x < a/6$ is

$$\int_0^{a/5} |\Psi_3(x)|^2 \, dx = \int_0^{a/6} \frac{2}{a} \sin^2\left(\frac{3\pi}{a}x\right) dx = \frac{2}{a}\int_0^{a/6}\left[0.5 - 0.5\cos\left(\frac{6\pi}{a}x\right)\right]dx$$

$$= \frac{1}{a}\int_0^{a/6}\left[1-\cos\left(\frac{6\pi}{a}x\right)\right]dx = \frac{1}{a}\left[x - \sin\left(\frac{6\pi}{a}x\right)/\left(\frac{6\pi}{a}\right)\right]\Bigg|_0^{a/6}$$

$$= \frac{1}{a}\left[\frac{a}{6} - a\sin\left(\frac{6\pi}{a}\frac{a}{6}\right)/(6\pi) - 0 + 0\right] = \frac{1}{6} - \sin(\pi)/(6\pi)$$

$$= \underline{0.16666667}$$

So, the % probability is **16.6667 %.**

	wave number	KE	probability density function
State 1	**$k_1 = 4.90874 \times 10^9$ rad/m**	**1.22406 eV**	**$3.125 \times 10^9 \sin^2(4.90874 \times 10^9 x)$ m^{-1}**
State 2	**$k_2 = 9.81748 \times 10^9$ rad/m**	**4.89623 eV**	**$3.125 \times 10^9 \sin^2(9.81748 \times 10^9 x)$ m^{-1}**
State 3	**$k_3 = 1.47262 \times 10^{10}$ rad/m**	**11.0165 eV**	**$3.125 \times 10^9 \sin^2(1.47262 \times 10^{10} x)$ m^{-1}**

State 3 Probability ($0 < x < a/6$) = **16.6667 %**