EE 362 Electronic, Magnetic, & Optical Properties of Materials
Examination #1 (Spring 2025)

Name **KEY B**

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class & show all work for credit. Insert equation sheet in exam. Answers should have 4-5 significant figures (use more for constants).

1) Copper (Cu) has a face-centered cubic crystal structure with a lattice constant of 3.615 Å. Find the atomic weight, number of atoms per unit cell, and unit cell volume (cm^3) for copper. Next, compute the atomic volume density avd_{Cu} (#/cm^3) and mass density md_{Cu} (g/cm^3) of copper.

From periodic table, the atomic weight of copper is $\mathbf{A_{r,Cu} = 63.546}$.

atoms/unit cell = 8 corners (1/8 atoms/corner) + 6 faces (1/2 atom/face) = **4 atoms**.

volume = $a^3 = (3.615\times10^{-10})^3 = 4.7241633\times10^{-29}$ m^3 ⇒ **volume = 7.72416×10^{-23} cm^3**.

avd_{Sr} = (# atoms/unit cell)/volume = $4 / (3.615\times10^{-10})^3 = 8.46711 \times 10^{28}$ atoms/m^3

⇒ **$avd_{Cu} = 8.46711 \times 10^{22}$ atoms/cm^3**.

The mass density strontium is

$md_{Cu} = (avd_{Cu})\,(A_{r,Cu}) / N_A = 8.46711 \times 10^{22}$ atoms/cm^3 $(63.546)/ 6.02214076 \times 10^{23}$

⇒ **$md_{Cu} = 8.93454$ g/cm^3**.

atomic weight = **63.546** # of atoms/unit cell = **4** volume = **7.72416×10^{-23} cm^3**

avd_{Cu} = **8.4671×10^{22} atoms/cm^3** md_{Cu} = **8.9345 g/cm^3**

2) An electron, near the surface of lithium (work function 2.93 eV), is simultaneously struck at one point by photons A ($\lambda_A = 680$ nm) and B ($\lambda_B = 560$ nm). First, find the frequency (Hz), momentum (kg•m/s), and kinetic energy (J and eV) of each photon. Assuming 100% energy transfer from the photons to the electron, will there be a photoelectron? If so, determine the maximum kinetic energy (J and eV), momentum (kg•m/s), velocity (m/s), and de Broglie wavelength (Å) of the photoelectron.

Photon A:

$$\nu = f = c/\lambda = 2.99792458 \times 10^8 / 680 \times 10^{-9} \Rightarrow \underline{\mathbf{\nu_A = f_A = 4.40871 \times 10^{14}\ Hz}}.$$

(2.2) $$p = h/\lambda = 6.62607015 \times 10^{-34} / 680 \times 10^{-9} \Rightarrow \underline{\mathbf{p_A = 9.74422 \times 10^{-28}\ kg{\cdot}m/s}}.$$

(text p. 27) $$KE_{\underline{A}} = h\nu_A = 6.62607015\times10^{-34}(4.408713\times10^{14})$$

$$\Rightarrow \underline{\mathbf{KE_A = 2.921244 \times 10^{-19}\ J = 1.8233\ eV}}.$$

Photon B:

$$\nu = f = c/\lambda = 2.99792458 \times 10^8 / 560 \times 10^{-9} \Rightarrow \underline{\mathbf{\nu_B = f_B = 5.353436 \times 10^{14}\ Hz}}.$$

(2.2) $$p = h/\lambda = 6.62607015 \times 10^{-34} / 560 \times 10^{-9} \Rightarrow \underline{\mathbf{p_B = 1.183227 \times 10^{-27}\ kg{\cdot}m/s}}.$$

(text p. 27) $$KE_B = h\nu_B = 6.62607015\times10^{-34}(5.353436\times10^{14})$$

$$\Rightarrow \underline{\mathbf{KE_B = 3.547225 \times 10^{-19}\ J = 2.2140\ eV}}.$$

Photoelectron:

(2.1) $$KE = 0.5mv^2 = h\nu - \Phi = (1.8233 + 2.214) - 2.93$$

$$\Rightarrow \underline{\mathbf{KE_e = 1.1073\ eV = 1.77409 \times 10^{-19}\ J}}.$$

$$KE = 0.5mv^2 \Rightarrow v_e = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(1.77409\times10^{-19})}{9.1093837\times10^{-31}}} \Rightarrow \underline{\mathbf{v_e = 624{,}106.0\ m/s}}.$$

$$p_e = mv = (9.1093837 \times 10^{-31})\,624{,}106 \Rightarrow \underline{\mathbf{p_e = 5.68522 \times 10^{-25}\ kg{\cdot}m/s}}$$

(2.3) $$\lambda_e = h/p_e = 6.62607015\times10^{-34} / 5.68522\times10^{-25} = 1.16549 \times 10^{-9}\ m$$

$$\Rightarrow \underline{\mathbf{\lambda_e = 11.6549\ Å}}.$$

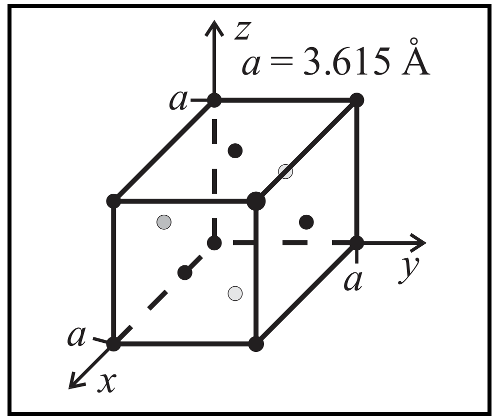
A: f_A = **4.4087 × 10^{14} Hz** p_A = **9.7442 × 10^{-28} kg·m/s** KE_A = **2.9212 × 10^{-19} J = 1.8233 eV**

B: f_B = **5.3534 × 10^{14} Hz** p_B = **1.1832 × 10^{-27} kg·m/s** KE_B = **3.5472 × 10^{-19} J = 2.2140 eV**

photoelectron? **[Yes]** / No (circle correct answer) KE_e = **1.7741 × 10^{-19} J** = **1.1073 eV**

e^-: mom_e = **5.68522 × 10^{-25} kg·m/s** $vel._e$ = **624,106 m/s** λ_e = **11.6549 Å**

3) Copper (Cu) has a face-centered cubic crystal structure with a lattice constant of 3.615 Å. Sketch the lattice structure in box provided (align to Cartesian axes w/ back left corner at origin). Next, find the number of atoms, Miller indices, and atomic surface density *asd* (#/cm^2) for the top surface as well as the diagonal surface connecting the left top and right bottom edges.



Top

atoms/top surface = 4 corners (1/4 atom/corner) + 1 face (1 atom/face) = **2 atoms**.

Miller indices

- x-intercept is ∞, y-intercept is ∞, & z-intercept is a.
- triplet $(1/\infty, 1/\infty, 1/a) = (0, 0, 1/a)$
- $a\,(0, 0, 1/a) = (0, 0, 1)$ $\Rightarrow$ **(0 0 1)**

area = $a^2 = (3.615\times10^{-10})^2 = 1.3068225\times10^{-19}$ m^2

asd = (# atoms/top)/area = $2 / (3.615\times10^{-10})^2 = 1.53043 \times 10^{19}$ atoms/m^2

$\Rightarrow$ ***asd* = 1.53043 × 10^{15} atoms/cm^2**.

Diagonal

atoms/diagonal surface = 4 corners (1/4 atom/corner) + 2 faces (1/2 atom/face) = **2 atoms**.

Miller indices

- x-intercept is ∞, y-intercept is a, & z-intercept is a.
- triplet $(1/\infty, 1/a, 1/a) = (0, 1/a, 1/a)$
- $a\,(0, 1/a, 1/a) = (0, 1, 1)$ $\Rightarrow$ **(0 1 1)**

area = $a\,(a^2 + a^2)^{0.5} = \sqrt{2}a^2 = \sqrt{2}\,(3.615\times10^{-10})^2 = 1.848126\times10^{-19}$ m^2

asd = (# atoms/top)/area = $2 / 1.848126\times10^{-19} = 1.082177 \times 10^{19}$ atoms/m^2

$\Rightarrow$ ***asd* = 1.082177 × 10^{15} atoms/cm^2**.

Top: # atoms = **2** Miller indices = **(0 0 1)** *asd* = **1.53043 × 10^{15} atoms/cm^2**

Diag: # atoms = **2** Miller indices = **(0 1 1)** *asd* = **1.08218 × 10^{15} atoms/cm^2**

4) An electron is confined in a 1D infinite potential well where $V = 0$ for $0 < x < a = 2.8$ Å and $V \to \infty$ elsewhere. For the first two possible states, find the wave number, kinetic energy (eV & J), and probability density function of the confined electron as well as the probability (%) of finding the electron in $0 < x < a/4$. [Hint: $\sin^2(A) = 0.5 - 0.5\cos(2A)$ and $\cos^2(A) = 0.5 + 0.5\cos(2A)$.]

Per (2.33), $k_n = n\pi/a$. For $n = 1$, $k_1 = \pi/2.8\times10^{-10}$ $\Rightarrow$ $\underline{\mathbf{k_1 = 1.1219975 \times 10^{10}\ rad/m}}$.

For $n = 2$, $k_2 = 2\pi/2.8\times10^{-10}$ $\Rightarrow$ $\underline{\mathbf{k_2 = 2.2439948 \times 10^{10}\ rad/m}}$.

Per (2.38), the kinetic energy

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054571817\times10^{-34})^2 \pi^2 n^2}{(2)9.1093837\times10^{-31}(2.8\times10^{-10})^2} = \frac{(1.054571817\times10^{-34})^2 \pi^2 n^2}{(2)9.1093837\times10^{-31}(2.8\times10^{-10})^2}$$

$$= 7.684524736\times10^{-19} n^2 \text{ (J)} = 4.796303 n^2 \text{ (eV)}$$

For $n = 1$, the kinetic energy is $\Rightarrow$ $\underline{\mathbf{E_1 = 7.68452 \times 10^{-19}\ J = 4.7963\ eV}}$.

For $n = 2$, the kinetic energy is $\Rightarrow$ $\underline{\mathbf{E_2 = 3.07381 \times 10^{-18}\ J = 19.18521\ eV}}$.

Per (2.36), the wave function is $\psi(x) = \sqrt{\frac{2}{a}}\sin(k_n x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$ m$^{-0.5}$ $n = 1, 2, 3\ldots.$

Per (2.17), $|\Psi(x,t)|^2 = |\psi(x)|^2 = \frac{2}{a}\sin^2\left(\frac{n\pi}{a}x\right) = \frac{2}{2.8\times10^{-10}}\sin^2\left(\frac{n\pi}{2.8\times10^{-10}}x\right)$.

For $n = 1$, the prob. density function is $\underline{\mathbf{|\Psi_1(x)|^2 = 7.14286 \times 10^9 \sin^2(1.122\times10^{10}x)\ m^{-1}}}$.

For $n = 2$, the prob. density function is $\underline{\mathbf{|\Psi_2(x)|^2 = 7.14286 \times 10^9 \sin^2(2.244\times10^{10}x)\ m^{-1}}}$.

$$\int_0^{a/4} |\Psi(x)|^2 dx = \int_0^{a/4} \frac{2}{a}\sin^2\left(\frac{n\pi}{a}x\right)dx = \frac{2}{a}\int_0^{a/4}\left[0.5 - 0.5\cos\left(\frac{2n\pi}{a}x\right)\right]dx$$

The probability is

$$= \frac{1}{a}\int_0^{a/4}\left[1 - \cos\left(\frac{2n\pi}{a}x\right)\right]dx = \frac{1}{a}\left[x - \sin\left(\frac{2n\pi}{a}x\right)/\left(\frac{2n\pi}{a}\right)\right]\Bigg|_0^{a/4}$$

$$= \frac{1}{a}\left[\frac{a}{4} - a\sin\left(\frac{2n\pi}{a}\frac{a}{4}\right)/(2n\pi)\right] = \underline{\frac{1}{4} - \sin\left(\frac{n\pi}{2}\right)/(2n\pi)}$$

For $n = 1$, $\int_0^{a/4} |\Psi_1(x)|^2 dx = \frac{1}{4} - \sin\left(\frac{\pi}{2}\right)/(2\pi) = \frac{1}{4} - \frac{1}{2\pi} = 0.09084506 = 9.084506\%$

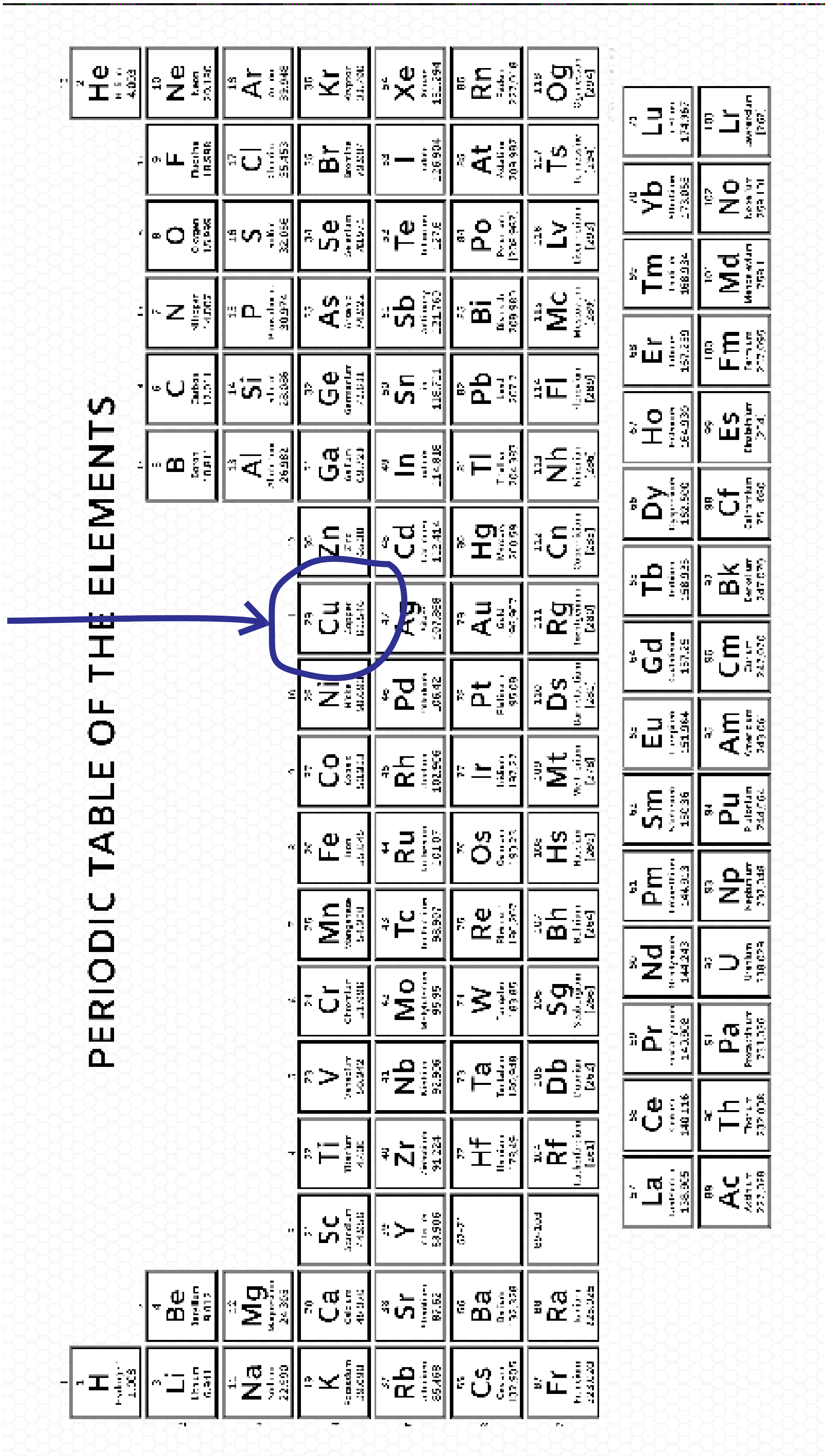
For $n = 2$, $\int_0^{a/4} |\Psi_2(x)|^2 dx = \frac{1}{4} - \sin\left(\frac{2\pi}{2}\right)/(2\pi) = \frac{1}{4} - 0 = 0.25 = 25\%$

State 1: wave number = $\underline{\mathbf{1.1220 \times 10^{10}\ rad/m}}$ KE = $\underline{\mathbf{4.7963\ eV}}$ = $\underline{\mathbf{7.68452 \times 10^{-19}\ J}}$

Prob. density function = $\underline{\mathbf{7.14286 \times 10^9 \sin^2(1.122\times10^{10}x)\ m^{-1}}}$ Prob'ty ($0 < x < a/4$) = $\underline{\mathbf{9.8045\ \%}}$

State 2: wave number = $\underline{\mathbf{2.2440 \times 10^{10}\ rad/m}}$ KE = $\underline{\mathbf{19.18521\ eV}}$ = $\underline{\mathbf{3.07381 \times 10^{-18}\ J}}$

Prob. density function = $\underline{\mathbf{7.14286 \times 10^9 \sin^2(2.244\times10^{10}x)\ m^{-1}}}$ Probability ($0 < x < a/4$) = $\underline{\mathbf{25\ \%}}$


PERIODIC TABLE OF THE ELEMENTS
H He
Li Be B C N O F Ne
Na Mg Al Si P S Cl Ar
K Ca Sc Ti V Cr Mn Fe Co Ni Cu Zn Ga Ge As Se Br Kr
Rb Sr Y Zr Nb Mo Tc Ru Rh Pd Ag Cd In Sn Sb Te I Xe
Cs Ba Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn
Fr Ra Rf Db Sg Bh Hs Mt Ds Rg Cn Nh Fl Mc Lv Ts Og
La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb Lu
Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No Lr