EE 362 Electronic, Magnetic, & Optical Properties of Materials
Examination #3 (Spring 2024)

Name KEY

Instructions: Closed book. Put answers in indicated spaces, use notation as given in class & show all work for credit. Insert equation sheet in exam. Answers should have 4-5 significant figures (use more for constants).

1) At thermal equilibrium @ 300 K, a germanium pn junction has a 0.43 V built-in potential. Assume the junction is uniformly doped on each side and has a cross sectional area of 2×10^{-8} m^2. If a third of the depletion layer in the p-region at zero bias, find the acceptor & donor doping concentrations (cm^{-3}), depletion layer thickness in the n- & p-regions (m), and junction capacitance (F).

Given $x_p = W/3$ where $W = x_p + x_n$. Therefore, $x_p = (x_p + x_n)/3$. Solving, we find $\mathbf{2x_p = x_n}$.

Per (7.17), $N_a x_p = N_d x_n = N_d (2x_p) \Rightarrow \mathbf{N_a / N_d = 2}$ **or** $\mathbf{N_a = 2N_d}$.

From Table B.4, $n_i = 2.4 \times 10^{13}$ cm^{-3} and $\varepsilon_r = 16.0$ for germanium at 300 K.

Per (7.10), $V_{bi} = 0.43 = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right) = \frac{8.617333 \bullet 10^{-7} \text{ eV/K (300 K)}}{e} \ln\left(\frac{(2N_d)N_d}{(2.4 \bullet 10^{-13})^2}\right)$

which leads to $N_d = \sqrt{\frac{(2.4 \bullet 10^{-13})^2}{2} e^{0.43/0.025852}} \Rightarrow \underline{N_d = 6.94284 \bullet 10^{16} \text{ cm}^{-3}}$.

Then, $N_a = 2N_d = 2(6.94284 \times 10^{16}$ cm$^{-3}) \Rightarrow \underline{\mathbf{N_a = 1.38857 \times 10^{17}\ cm^{-3}}}$.

Per (7.28),

$$x_n = \left\{\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a + N_d}\right\}^{1/2}$$

$$= \left\{\frac{2(16)8.8541878 \bullet 10^{-12}(0.43)}{1.602176634 \bullet 10^{-19}}(2)\frac{1}{1.38857 \bullet 10^{23} + 6.94284 \bullet 10^{22}}\right\}^{1/2}$$

$\Rightarrow \underline{\mathbf{x_n = 8.54504 \times 10^{-8}\ m}}$.

Then, $2x_p = x_n \Rightarrow x_p = x_n/2 = (8.54504 \times 10^{-8}) / 2 \Rightarrow \underline{\mathbf{x_p = 4.27252 \times 10^{-8}\ m}}$.

Depletion layer width is $W = x_p + x_n = 4.27252 \times 10^{-8} + 8.54504 \times 10^{-8} = 1.28176 \times 10^{-7}$ m.

Per (7.43), $C' = \varepsilon_s / W$ which, after multiplication by the junction area, leads to

$C = A\varepsilon_s / W = (2 \times 10^{-8})\, 16\, (8.8541878 \times 10^{-12}) / 1.28176 \times 10^{-7} \Rightarrow \underline{\mathbf{C = 22.1051\ pF}}$

acceptor doping conc. = $\underline{\mathbf{N_a = 1.3886 \times 10^{17}\ cm^{-3}}}$ donor doping conc. = $\underline{\mathbf{N_d = 6.9428 \times 10^{16}\ cm^{-3}}}$

depl. layer thickness$_{\text{n-reg}}$ = $\underline{\mathbf{x_n = 85.4504\ nm}}$ depl. layer thickness$_{\text{p-reg}}$ = $\underline{\mathbf{x_p = 42.7252\ nm}}$

junction capacitance = $\underline{\mathbf{C = 22.1051\ pF}}$

2) At thermal equilibrium in Greenland @ **250 K**, a silicon pn diode has the parameters given below. Assume the junction is uniformly doped on each side. For the p- & n-regions, find the thermal equilibrium electron & hole concentrations (cm^{-3}) as well as diffusion coefficients (cm^2/s) & lengths (m). Next, find the ideal reverse saturation current density (A/m^2) in the depletion layer. Then, find the current density (A/m^2) and current through the depletion layer when $V_a = 0.12$ V & 0.36 V.

$A = 8 \times 10^{-4}$ cm^2	$n_i = 10^{10}$ cm^{-3}	$\varepsilon_r = 11.8$	$E_g = 1.12$ eV
p-region	$N_a = 2 \times 10^{16}$ cm^{-3}	$\mu_p = 190$ cm^2/V-s	$\tau_{p0} = 90$ ns
n-region	$N_d = 6 \times 10^{15}$ cm^{-3}	$\mu_n = 500$ cm^2/V-s	$\tau_{n0} = 240$ ns

p-region

$p_{p0} \approx N_a \Rightarrow$ **$p_{p0} = 2 \times 10^{16}$ cm^{-3}**.

Using (4.43), $n_{p0} = n_i^2/p_{p0} = (10^{10})^2/2 \times 10^{16} \Rightarrow$ **$n_{p0} = 5 \times 10^3$ cm^{-3} = 5000 cm^{-3}**.

Using Einstein relation (5.47) with **$T = 250$ K**,

$$\frac{D_p}{\mu_p} = \frac{k_B T}{e} = V_t = \frac{8.617333 \bullet 10^{-7}\ \text{eV/K}\,(250\,\text{K})}{e} = 0.0215433\ \text{V}.$$

Therefore, $D_p = 0.0215433(\mu_p) = 0.0215433(190) \Rightarrow$ **$D_p = 4.093233$ cm^2/s**.

Per (8.9), $L_p = \sqrt{D_p \tau_{p0}} = \sqrt{4.093233 \bullet 10^{-4}(90 \bullet 10^{-9})} \Rightarrow$ **$L_p = 6.06952 \times 10^{-6}$ m**.

n-region

$n_{n0} \approx N_d \Rightarrow$ **$n_{n0} = 6 \times 10^{15}$ cm^{-3}**.

Using (4.43), $p_{n0} = n_i^2/n_{n0} = (10^{10})^2/6 \times 10^{15} \Rightarrow$ **$p_{n0} = 1.667 \times 10^4$ cm^{-3} = 16,666.7 cm^{-3}**.

Using Einstein relation (5.47), $\dfrac{D_n}{\mu_n} = \dfrac{k_B T}{e} = 0.0215433$ V.

Therefore, $D_n = 0.0215433(\mu_n) = 0.0215433(500) \Rightarrow$ **$D_n = 10.771667$ cm^2/s**.

Per (8.9), $L_n = \sqrt{D_n \tau_{n0}} = \sqrt{10.771667 \bullet 10^{-4}(240 \bullet 10^{-9})} \Rightarrow$ **$L_n = 1.60786 \times 10^{-5}$ m**.

Per (8.26),

$$J_s = \frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} = 1.6021766 \bullet 10^{-19}\left[\frac{4.09323 \bullet 10^{-4}(1.667 \bullet 10^{10})}{6.06952 \bullet 10^{-6}} + \frac{10.7717 \bullet 10^{-4}(5 \bullet 10^{9})}{1.60786 \bullet 10^{-5}}\right]$$

$\Rightarrow$ **$J_s = 2.337504 \times 10^{-7}$ A/m^2**.

Per (8.27), $J = J_s\left[e^{V_a/V_t} - 1\right] = 2.337504 \bullet 10^{-7}\left[e^{V_a/0.0215433} - 1\right]$ and $I = J(A)$.

V_a = 0.12 V

$$J = 2.337504 \bullet 10^{-7}\left[e^{0.12/0.0215433} - 1\right] \Rightarrow \underline{J\,(0.12\text{ V}) = 6.112069 \times 10^{-5}\text{ A/m}^2}.$$

$$I = J(A) = 6.11207 \times 10^{-5}\,(8 \times 10^{-8}) \Rightarrow \underline{I\,(0.12\text{ V}) = 4.889655 \times 10^{-12}\text{ A}}.$$

V_a = 0.36 V

$$J = 2.337504 \bullet 10^{-7}\left[e^{0.36/0.0215433} - 1\right] \Rightarrow \underline{J\,(0.36\text{ V}) = 4.227007\text{ A/m}^2}.$$

$$I = J(A) = 4.227007\,(8 \times 10^{-8}) \Rightarrow \underline{I\,(0.36\text{ V}) = 3.38161 \times 10^{-7}\text{ A}}.$$

p-region:

$p_{p0} = 2 \times 10^{16}\text{ cm}^{-3}$ **$n_{p0} = 5 \times 10^{3}\text{ cm}^{-3}$** **$D_p = 4.0932\text{ cm}^2/\text{s}$** **$L_p = 6.0695 \times 10^{-6}\text{ m}$**

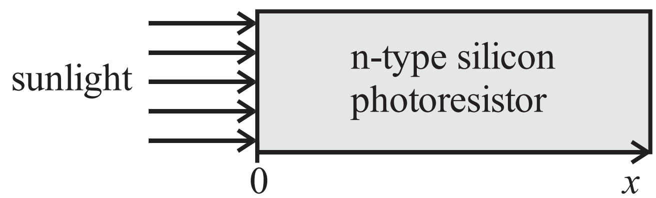
n-region:

$n_{n0} = 6 \times 10^{15}\text{ cm}^{-3}$ **$p_{n0} = 1.667 \times 10^{4}\text{ cm}^{-3}$** **$D_n = 10.7717\text{ cm}^2/\text{s}$** **$L_n = 1.60786 \times 10^{-5}\text{ m}$**

$J_s = 2.3375 \times 10^{-7}\text{ A/m}^2$ $J(0.12\text{ V}) =$ **$6.11207 \times 10^{-5}\text{ A/m}^2$** $J(0.36\text{ V}) =$ **4.22701 A/m^2**

$I(0.12\text{ V}) =$ **$4.889655 \times 10^{-12}\text{ A} = 4.8897\text{ pA}$** $I(0.36\text{ V}) =$ **$3.38161 \times 10^{-7}\text{ A} = 338.161\text{ nA}$**

3) An **n-type** silicon photoresistor at 300 K is doped to a donor concentration of 6×10^{15} cm^{-3}. Assume that $\mu_n = 1160$ cm^2/V-s, $\mu_p = 410$ cm^2/V-s, $\tau_{n0} = 1$ µs, and $\tau_{p0} = 0.5$ µs for this silicon and that no external electric field is applied. At steady-state, sunlight striking the photoresistor generates an excess carrier concentration of 1.5×10^{14} cm^{-3} at the surface. Find the ambipolar mobility (cm^2/V-s) and diffusion coefficient (cm^2/s). Neglecting surface effects, write the appropriate **general** ambipolar transport equation for the **minority** carriers for $x > 0$. After applying the conditions under which the photoresistor is operating, write the **particular** ambipolar transport equation for $x > 0$. Then, find the excess minority charge carrier concentration (cm^{-3}) and hole diffusion current density (A/cm^2) as a function of distance x into the photoresistor.



From Table B.4, $n_i = 1.5 \times 10^{10}$ cm^{-3} = 1.5×10^{16} m^{-3} and $\varepsilon_r = 11.7$ for silicon at 300 K.

Per (7.10), $V_t = \dfrac{8.617333 \bullet 10^{-7}\text{ eV/K}(300\text{K})}{e} = 0.025852\text{ V}$

n-type silicon (majority carriers are electrons)

$n_0 \approx N_d \quad \Rightarrow \quad$ **$n_0 = 6 \times 10^{15}$ cm^{-3}**.

Using (4.43), $p_0 = n_i^2/n_0 = (1.5 \times 10^{10})^2/6 \times 10^{15} \Rightarrow$ **$p_0 = 3.75 \times 10^4$ cm^{-3} = 37,500 cm^{-3}**.

Using Einstein relation (5.47), $\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = \dfrac{k_B T}{e} = 0.025852\text{ V}$.

Therefore, for the **minority** charge carriers (holes)

$D_p = 0.025852(\mu_p) = 0.025852(410) \quad \Rightarrow \quad$ **$D_p = 10.59932$ cm^2/s**.

Per (8.9), $L_p = \sqrt{D_p \tau_{p0}} = \sqrt{10.59932 \bullet 10^{-4}(0.5 \bullet 10^{-6})} \quad \Rightarrow \quad$ **$L_p = 2.3021 \times 10^{-5}$ m**.

Since $p_{n0} << n_{n0}$ and $\delta n << n_0$, use (6.47) to get $D' = D_p \quad \Rightarrow \quad$ **$D' = 10.59932$ cm^2/s**.

Since $p_{n0} << n_{n0}$ and $\delta n << n_0$, use (6.48) to get $\mu' = -\mu_p \quad \Rightarrow \quad$ **$\mu' = -410$ cm^2/V-s.**

Per (6.56), for n-type semiconductor, the general ambipolar transport equation is

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}.$$

Here the particular ambipolar transport equation for $x > 0$ where $E = 0$, $g' = 0$ & we have steady-state, is

$$0 = D_p \frac{\partial^2(\delta p)}{\partial x^2} - 0 + 0 - \frac{\delta p}{\tau_{p0}} \text{ which reduces to } \underline{\frac{\partial^2(\delta p)}{\partial x^2} - \frac{\delta p}{D_p \tau_{p0}} = \frac{\partial^2(\delta p)}{\partial x^2} - \frac{\delta p}{L_p{}^2} = 0}.$$

The general solution for this wave equation is $\delta p(x) = Ae^{-x/L_p} + Be^{x/L_p}$.

Applying the boundary conditions that $\delta p(x)$ is finite (i.e., $B = 0$) for $x > 0$, and, at the surface, $\delta p(x = 0) = 1.5 \times 10^{14}\ \text{cm}^{-3} = A$ leads to the particular solution

$$\underline{\delta p(x) = 1.5 \bullet 10^{14} e^{-x/2.3021 \bullet 10^{-5}}\ \text{cm}^{-3}\quad \text{for } x \geq 0}.$$

Using MKS units, the minority carrier hole diffusion current density is found using (5.34)-

$$J_{px|dif} = -eD_p \frac{dp}{dx} = -eD_p \frac{d(\delta p(x))}{dx}$$

$$= -1.6021766 \bullet 10^{-19} (10.59932 \bullet 10^{-4}) \frac{-1.5 \bullet 10^{20}}{2.3021 \bullet 10^{-5}} e^{-x/2.3021 \bullet 10^{-5}}$$

$$= 1105.8714 e^{-x/2.3021 \bullet 10^{-5}}\ \text{A/m}^2 \quad \text{for } x \geq 0$$

Converting the units to A/cm^2, we get $\underline{J_{px|dif} = 0.11058714 e^{-x/2.3021 \bullet 10^{-5}}\ \text{A/cm}^2 \quad \text{for } x \geq 0}$.

ambipolar mobility = ***μ' = -410 cm²/V-s***   ambipolar diff. coeff. = ***D' = 10.5993 cm²/s***

general ambipolar transport eq'n = $\underline{\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}}$

particular ambipolar transport eq'n = $\underline{\frac{\partial^2(\delta p)}{\partial x^2} - \frac{\delta p}{D_p \tau_{p0}} = \frac{\partial^2(\delta p)}{\partial x^2} - \frac{\delta p}{L_p{}^2} = 0}$

excess minority charge carrier conc. = $\underline{\delta p(x) = 1.5 \bullet 10^{14} e^{-x/2.3021 \bullet 10^{-5}}\ \text{cm}^{-3} \quad \text{for } x \geq 0}$

hole diffusion current density = $\underline{J_{px|dif} = 0.11059 e^{-x/2.3021 \bullet 10^{-5}}\ \text{A/cm}^2 \quad \text{for } x \geq 0}$