

a) For a one-sided pn⁺ junction where $N_d \gg N_a$, find approximate expressions for W , x_n , x_p , and C' . b) A silicon pn⁺ junction at 300 K has a cross-sectional area $8 \times 10^{-5} \text{ cm}^2$, $N_d = 8 \times 10^{16} \text{ cm}^{-3}$, and $N_a = 10^{15} \text{ cm}^{-3}$. Find *i*) V_{bi} , *ii*) W , x_n , x_p , & the junction capacitance with $V_R = 0$, and *iii*) W , x_n , x_p , & the junction capacitance with $V_R = 1 \text{ V}$.

a)

$$\text{Per (7.34), } W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right\}^{1/2} = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_a / N_d + 1}{N_a} \right) \right\}^{1/2} \Bigg|_{N_d \gg N_a}$$

$$\Rightarrow W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_a} \right\}^{1/2}$$

Per (7.28) with V_{bi} replaced w/ $V_{\text{tot}} = V_{bi} + V_R$,

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \frac{1}{N_a + N_d} \right\}^{1/2} = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1 / N_d}{N_a / N_d + 1} \right) \right\}^{1/2} \Bigg|_{N_d \gg N_a}$$

$$\Rightarrow \underline{x_n \approx 0.}$$

Per (7.29) with V_{bi} replaced w/ $V_{\text{tot}} = V_{bi} + V_R$,

$$x_p = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_d}{N_a} \right) \frac{1}{N_a + N_d} \right\}^{1/2} = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{1}{N_a} \right) \left(\frac{1}{N_a / N_d + 1} \right) \right\}^{1/2} \Bigg|_{N_d \gg N_a}$$

$$\Rightarrow x_p \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_a} \right\}^{1/2} \approx W.$$

$$\text{Per (7.43), } C' = \frac{\epsilon_s}{W} \approx \epsilon_s \left\{ \frac{e N_a}{2\epsilon_s(V_{bi} + V_R)} \right\}^{1/2} \Rightarrow C' \approx \left\{ \frac{e \epsilon_s N_a}{2(V_{bi} + V_R)} \right\}^{1/2}$$

b) From Table B.4, $\epsilon_r = 11.7$, $n_i = 1.5 \times 10^{10} \text{ cm}^{-3} = 1.5 \times 10^{16} \text{ m}^{-3}$ for silicon at 300 K.

$$i) \text{ Per (7.10), the built-in voltage is } V_{bi} = \frac{k_B T}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right).$$

$$\text{At 300 K, the thermal voltage } V_t = \frac{k_B T}{e} = \frac{8.617333 \times 10^{-5} \text{ eV/K (300K)}}{e} = 0.025852 \text{ V.}$$

$$\text{Here, } V_{bi} = 0.025852 \ln \left(\frac{10^{15} (8 \times 10^{16})}{(1.5 \times 10^{10})^2} \right) \Rightarrow \underline{V_{bi} = 0.687584 \text{ V.}}$$

ii) $V_R = 0$

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_a} \right\}^{1/2} = \left\{ \frac{2(11.7)8.8541878 \times 10^{-12}(0.687584 + 0)}{1.602176634 \times 10^{-19}(1 \times 10^{21})} \right\}^{1/2}$$

$$\Rightarrow \underline{W \approx 9.42953 \times 10^{-7} \text{ m} = 942.953 \text{ nm.}}$$

$$\Rightarrow \underline{x_n \approx 0.}$$

$$\Rightarrow \underline{x_p \approx W \approx 9.42953 \times 10^{-7} \text{ m} = 942.953 \text{ nm.}}$$

$$\text{Per (7.43), } C' = \frac{\epsilon_s}{W} \approx \frac{(11.7)8.8541878 \times 10^{-12}}{9.42953 \times 10^{-7}} \approx 1.09861 \times 10^{-4} \text{ F/m}^2.$$

$$C = C'A = 1.09861 \times 10^{-4} \text{ F/m}^2 (8 \times 10^{-9} \text{ m}^2) \Rightarrow \underline{C \approx 8.7889 \times 10^{-13} \text{ F} = 0.8789 \text{ pF.}}$$

Bonus The exact values are:

$$W = 948.828 \text{ nm}, x_n = 11.714 \text{ nm}, x_p = 937.114 \text{ nm}, \& C = 0.8734 \text{ pF. Error is } \sim 0.6\%.$$

iii) $V_R = 1 \text{ V}$

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_a} \right\}^{1/2} = \left\{ \frac{2(11.7)8.8541878 \times 10^{-12}(0.687584 + 1)}{1.602176634 \times 10^{-19}(1 \times 10^{21})} \right\}^{1/2}$$

$$\Rightarrow \underline{W \approx 1.47727 \times 10^{-6} \text{ m} = 1477.27 \text{ nm} = 1.47727 \text{ } \mu\text{m.}}$$

$$\Rightarrow \underline{x_n \approx 0.}$$

$$\Rightarrow \underline{x_p \approx W \approx 1.47727 \times 10^{-6} \text{ m} = 1477.27 \text{ nm} = 1.47727 \text{ } \mu\text{m.}}$$

$$\text{Per (7.43), } C' = \frac{\epsilon_s}{W} \approx \frac{(11.7)8.8541878 \times 10^{-12}}{1.47727 \times 10^{-6}} \approx 7.01253 \times 10^{-5} \text{ F/m}^2.$$

$$C = C'A = 7.01253 \times 10^{-5} \text{ F/m}^2 (8 \times 10^{-9} \text{ m}^2) \Rightarrow \underline{C \approx 5.610024 \times 10^{-13} \text{ F} = 0.5610 \text{ pF.}}$$

Bonus The exact values are:

$$W = 1486.474 \text{ nm}, x_n = 18.351 \text{ nm}, x_p = 1468.123 \text{ nm}, \& C = 0.5575 \text{ pF. Error is } \sim 0.6\%.$$