For a uniformly doped ($N_a = 8 \times 10^{15}$ cm^{-3} on the p-side and $N_d = 3 \times 10^{16}$ cm^{-3} on the n-side) Germanium pn junction at 300 K with cross-sectional area 40×10^{-9} m^2, calculate x_n, x_p, W, $|E_{\max}|$, C', and C when: a) $V_R = 0$ and b) $V_R = 1$ V.

From Table B.4, $\varepsilon_r = 16.0$, $n_i = 2.4 \times 10^{13}$ cm^{-3} $= 2.4 \times 10^{19}$ m^{-3} for Ge at 300 K.

Per (7.10), the built-in voltage is $V_{bi} = \dfrac{k_B T}{e} \ln\left(\dfrac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$.

At 300 K, the thermal voltage $V_t = \dfrac{k_B T}{e} = \dfrac{8.617333 \times 10^{-5}\ \text{eV/K}\,(300\ \text{K})}{e} = 0.025852$ V.

Here, $V_{bi} = 0.025852 \ln\left(\dfrac{8 \times 10^{15}(3 \times 10^{16})}{(2.4 \times 10^{13})^2}\right)$ $\Rightarrow$ $\boldsymbol{V_{bi} = 0.334526}$ **V**.

a) $V_R = 0$

Per (7.28),

$$x_n = \left\{\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a + N_d}\right\}^{1/2} = \left\{\frac{2(16)8.8542 \times 10^{-12}(0.33453)}{1.602176634 \times 10^{-19}}\left(\frac{8}{30}\right)\frac{1}{8 \times 10^{21} + 3 \times 10^{22}}\right\}^{1/2}$$

$\Rightarrow$ **$x_n = 6.4432 \times 10^{-8}$ m = 64.432 nm**.

Per (7.29),

$$x_p = \left\{\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\frac{1}{N_a + N_d}\right\}^{1/2} = \left\{\frac{2(16)8.8542 \times 10^{-12}(0.33453)}{1.602176634 \times 10^{-19}}\left(\frac{30}{8}\right)\frac{1}{8 \times 10^{21} + 3 \times 10^{22}}\right\}^{1/2}$$

$\Rightarrow$ **$x_p = 2.4162 \times 10^{-7}$ m = 241.62 nm**.

Per (7.30), $W = x_n + x_p = 64.432 + 241.62$ $\Rightarrow$ **$W = 3.06052 \times 10^{-7}$ m = 306.052 nm**.

Per (7.37),

$$|E_{\max}| = \left|\frac{-2(V_{bi} + V_R)}{W}\right| = \frac{2(0.334526 + 0)}{3.06052 \times 10^{-7}}$$

$\Rightarrow$ **$|E_{max}| = 2.1861 \times 10^6$ V/m = 2.1861 MV/m**.

Per (7.43), $C' = \dfrac{\varepsilon_s}{W} = \dfrac{16(8.8541878 \times 10^{-12})}{3.06052 \times 10^{-7}}$ $\Rightarrow$ **$C' = 4.62885 \times 10^{-4}$ F/m^2**.

$C = C'A = 4.62885 \times 10^{-4}$ F/m^2 $(40 \times 10^{-9}$ m$^2)$ $\Rightarrow$ **$C = 1.85154 \times 10^{-11}$ F = 18.5154 pF**.

b) $\boxed{V_R = 1\text{ V}}$

Replace V_{bi} w/ $V_{\text{tot}} = V_{bi} + V_R = 0.334526 + 1 = 1.334526$ V in prior equations.

$$x_n = \left\{ \frac{2(16)8.8542\times10^{-12}(1.334526)}{1.602176634\times10^{-19}} \left(\frac{8}{30}\right) \frac{1}{8\times10^{21}+3\times10^{22}} \right\}^{1/2}$$

$\Rightarrow$ **$x_n = 1.28692\times10^{-7}$ m = 128.692 nm**.

$$x_p = \left\{ \frac{2(16)8.8542\times10^{-12}(1.334526)}{1.602176634\times10^{-19}} \left(\frac{30}{8}\right) \frac{1}{8\times10^{21}+3\times10^{22}} \right\}^{1/2}$$

$\Rightarrow$ **$x_p = 4.82594\times10^{-7}$ m = 482.594 nm**.

Per (7.30), $W = x_n + x_p = 128.692 + 482.594$ $\Rightarrow$ **$W = 6.11285\times10^{-7}$ m = 611.285 nm**.

Per (7.37),

$$|E_{\max}| = \left| \frac{-2(V_{bi}+V_R)}{W} \right| = \frac{2(1.334526)}{6.11285\times10^{-7}}$$

$\Rightarrow$ **$|E_{max}| = 4.3663\times10^{6}$ V/m = 4.3663 MV/m**.

Per (7.43), $C' = \frac{\varepsilon_s}{W} = \frac{16(8.8541878\times10^{-12})}{6.11285\times10^{-7}}$ $\Rightarrow$ **$C' = 2.31753\times10^{-4}$ F/m^2**.

$C = C'A = 2.31753\times10^{-4}$ F/m^2 $(40\times10^{-9}$ m$^2)$ $\Rightarrow$ **$C = 9.27011\times10^{-12}$ F = 9.27011 pF**.